

Hybrid Ray–Wave Modeling Of Power Propagation In Dual-Core Optical Fibers

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Résumé – Le développement rapide des systèmes de communication optique à haute capacité a suscité un intérêt croissant pour les fibres optiques à double cœur (DCOF) en raison de leur potentiel pour un débit de données élevé et une réduction de la diaphonie. La modélisation précise de la propagation de la puissance dans de telles fibres reste un défi, car les méthodes traditionnelles de traçage de rayons, bien que rapides sur le plan computationnel, ne capturent pas les effets d'interférence des modes, tandis que les approches basées sur la théorie des ondes fournissent des distributions de champ détaillées mais sont coûteuses en temps de calcul pour des fibres longues ou des géométries complexes. Dans cette étude, nous proposons une approche hybride Ray–Wave combinant les avantages des deux méthodes pour simuler avec précision la propagation de la puissance dans les fibres à double cœur. Le modèle hybride utilise d'abord le traçage de rayons pour simuler la trajectoire des rayons lumineux, en tenant compte des réflexions totales internes et de la dispersion géométrique. Ces résultats servent ensuite à initialiser des simulations basées sur les ondes pour un calcul détaillé du couplage des modes entre cœurs et des motifs d'interférence. La méthode a été appliquée à une fibre standard à double cœur avec des diamètres de cœur de 8 μm , une séparation cœur-à-cœur de 25 μm et un diamètre de gaine de 125 μm , opérant à des longueurs d'onde de 850 nm, 1310 nm et 1550 nm. Les simulations ont montré que l'approche hybride prédit avec précision le transfert de puissance entre les cœurs, avec une diaphonie maximale inter-cœur de -23 dB à 1550 nm, conforme aux attentes théoriques. En comparaison, le traçage de rayons surestimait la fuite de puissance jusqu'à 12 %, tandis que les simulations uniquement basées sur les ondes nécessitaient environ 4 fois plus de temps de calcul pour des fibres de plus de 1 km. La validation a été réalisée par une comparaison croisée avec des modèles conventionnels, montrant que la méthode hybride réduit le temps de calcul d'environ 65 % tout en maintenant une déviation inférieure à 5 % dans la distribution de puissance prédite. Les résultats indiquent également que l'approche hybride est particulièrement efficace dans les régimes où la fréquence normalisée (*V*-number) dépasse 2,4, correspondant à des modes faiblement couplés dans chaque cœur. Ce travail fournit un cadre pratique et efficace pour la modélisation de la propagation dans les fibres à double cœur, comblant le fossé entre l'optique géométrique et l'optique ondulatoire. Il permet aux concepteurs de prédire avec grande précision le comportement modal, la diaphonie inter-cœur et la dispersion tout en minimisant les ressources computationnelles. L'approche est facilement extensible aux systèmes de fibres multi-cœurs et peut être intégrée aux mesures expérimentales pour une validation de conception améliorée. Globalement, la méthode hybride Ray–Wave représente un outil prometteur pour les systèmes de communication optique de prochaine génération, offrant à la fois rapidité et précision.

Mots-clés : Fibres optiques à double cœur, Modélisation hybride, Traçage de rayons, Théorie des ondes, Couplage modal

Abstract– The rapid development of high-capacity optical communication systems has led to increasing interest in dual-core optical fibers (DCOFs) due to their potential for high data throughput and reduced crosstalk. Accurate modeling of power propagation in such fibers remains challenging because traditional ray-tracing methods, while computationally efficient, fail to capture modal interference effects, whereas wave theory approaches provide detailed field distributions but are computationally expensive for long fibers or complex geometries. In this study, we propose a hybrid Ray–Wave modeling approach that combines the strengths of both methods to accurately simulate power propagation in dual-core fibers. The hybrid model first employs ray-tracing to simulate the trajectory of light rays, accounting for total internal reflections and geometric dispersion. These results are then used to initialize wave-based simulations for detailed computation of inter-core mode coupling and interference patterns. The method was applied to a standard dual-core fiber with

core diameters of 8 μm , core-to-core separation of 25 μm , and a cladding diameter of 125 μm , operating at wavelengths of 850 nm, 1310 nm, and 1550 nm. Simulations demonstrated that the hybrid approach accurately predicts power transfer between cores, showing a maximum inter-core crosstalk of -23 dB at 1550 nm, consistent with theoretical expectations. In comparison, pure ray-tracing overestimated power leakage by up to 12%, while wave-only simulations required approximately $4\times$ more computation time for fibers longer than 1 km. Validation was performed through cross-comparison with conventional models, demonstrating that the hybrid method reduces computation time by approximately 65% while maintaining $<5\%$ deviation in predicted power distribution. The results also indicate that the hybrid approach is particularly effective in regimes where the normalized frequency (V-number) exceeds 2.4, corresponding to weakly coupled modes in each core. This work provides a practical and efficient framework for modeling dual-core fiber propagation, bridging the gap between geometric optics and wave optics. It enables designers to predict modal behavior, inter-core crosstalk, and dispersion with high accuracy while minimizing computational resources. The approach is easily extendable to multi-core fiber systems and can be integrated with experimental measurements for enhanced design validation. Overall, the hybrid Ray-Wave method represents a promising tool for next-generation fiber optic communication systems, offering both speed and precision.

Keywords: Dual-core optical fibers, Hybrid modeling, Ray-tracing, Wave theory, Mode coupling

I. INTRODUCTION

Accurate modeling of optical power propagation in dual-core fibers is essential for predicting critical performance metrics such as power distribution, attenuation, inter-core coupling length, and crosstalk, which typically ranges from -20 dB to -40 dB depending on core separation and operating wavelength [5][6]. Classical modeling approaches are mainly based on either ray-tracing techniques or wave theory formulations. Ray-tracing methods, derived from geometrical optics, are widely used due to their low computational cost, allowing simulations over fiber lengths exceeding $1 - 10$ km with modest numerical resources [7]. However, these approaches neglect wave phenomena such as interference and diffraction, leading to power prediction errors that may exceed $10 - 15\%$ in closely spaced dual-core configurations [8]. Wave theory-based approaches, including solutions of Maxwell's equations and coupled-mode theory, provide a rigorous description of electromagnetic field distributions and inter-core modal interactions [9][10]. These methods are particularly accurate for analyzing mode coupling coefficients, which typically lie in the range of $10^{-3} - 10^{-1} \text{ m}^{-1}$ for weakly coupled dual-core fibers [11]. Nevertheless, full-wave simulations are computationally demanding, with computation times increasing by a factor of 3 to 5 when fiber length exceeds 1 km or when broadband wavelength analysis is required [12]. To address these challenges, hybrid Ray-Wave modeling approaches have been proposed as an effective compromise between accuracy and efficiency [13]. By combining ray-tracing for global propagation behavior with wave-based calculations for local modal interactions, hybrid methods can reduce computation time by approximately $50 - 70\%$ while maintaining prediction errors below 5% for power distribution and crosstalk [14]. Despite these advantages, the application of hybrid modeling to dual-core optical fibers remains limited, particularly for systematic analysis of power propagation over standard telecommunication wavelengths (850 nm, 1310 nm, and 1550 nm) [15]. The objective of this work is therefore to develop and validate a hybrid Ray-Wave modeling framework for power propagation in dual-core optical fibers. The proposed approach integrates ray-tracing to efficiently model long-distance propagation with wave theory to accurately capture inter-core coupling and interference effects. This study aims to provide quantitative insights into the domain of validity of each method and to establish practical modeling recommendations for the design and optimization of next-generation dual-core fiber systems.

II. MATHEMATICAL MODELING HYBRID RAY

The propagation of optical power in dual-core optical fibers involves both geometrical and wave-optical phenomena, especially when the core separation is small and modal coupling becomes significant. To accurately describe these effects while maintaining computational efficiency, a hybrid modeling framework combining ray-tracing and wave theory is proposed.

2.1 Global Ray-Based Propagation Model

At the first stage, light propagation is described using a ray-tracing approach based on geometrical optics. Each ray is

characterized by its position (x_i, y_i) , direction vector, and optical power P_i . The ray trajectories inside each core are determined by Snell's law and total internal reflection conditions at the core-cladding interface. The power evolution along the propagation distance z is given by:

$$P_i(z) = P_i(0)\exp(-\alpha z) \quad (1)$$

Where:

- α represents the attenuation coefficient,

α is typically ranging from 0.2 to 0.35 dB·km⁻¹ for standard silica fibers at 1550 nm [1][2]. Ray-tracing efficiently captures macroscopic propagation behavior over distances exceeding 1 – 10 km, with negligible computational overhead [3].

2.2 Local Wave-Based Modal Analysis

While ray-tracing describes global energy transport, it does not capture modal interference or inter-core coupling. To address this limitation, a wave-based model is locally applied using coupled-mode theory. The evolution of modal amplitudes $A_1(z)$ and $A_2(z)$ in the two cores is described by:

$$\begin{aligned} \frac{dA_1}{dz} &= -j\beta_1 A_1 - j\kappa A_2 \\ \frac{dA_2}{dz} &= -j\beta_2 A_2 - j\kappa A_1 \end{aligned} \quad (2)$$

Where:

- β_1 and β_2 are the propagation constants of the individual cores
- and κ is the coupling coefficient

κ is typically in the range of $10^{-3} - 10^{-2} \text{ m}^{-1}$ for weakly coupled dual-core fibers [4][5]. This formulation enables accurate prediction of power exchange and beating length, which usually lies between 5 and 50 m depending on wavelength and core spacing [6].

2.3 Ray-Wave Coupling Strategy

The hybrid framework couples ray-tracing and wave theory by projecting the ray-based field distribution onto the modal basis of the wave model. The effective field at the input of the wave-based section is expressed as:

$$E_{eff}(x, y) = \sum_i \sqrt{P_i} \delta(x - x_i, y - y_i) \exp(j\phi_i) \quad (3)$$

Where:

- ϕ_i denotes the phase accumulated along the ray path.

The modal excitation coefficients are then obtained using overlap integrals:

$$A_m = \iint E_{eff}(x, y)\psi_m^*(x, y)dx dy \quad (4)$$

This approach preserves phase information while allowing efficient energy tracking over long distances. Numerical results indicate that this coupling reduces prediction errors in power distribution from >10% (ray-only) to <5% when compared with full-wave simulations [7].

2.4 Computational Efficiency and Stability

One of the main advantages of the hybrid framework is its computational efficiency. For a 1 km dual-core fiber, the hybrid model requires approximately 35 – 45% of the computation time needed by a full-wave simulation, while maintaining comparable accuracy in predicting power transfer and crosstalk [8]. The framework also remains numerically stable over broadband wavelength ranges (850 – 1550 nm), making it suitable for parametric optimization studies [9].

2.5 Domain of Validity

The hybrid modeling framework is particularly effective when the normalized frequency satisfies:

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} > 2.4 \quad (5)$$

Where:

- a is the core radius
- n_1, n_2 are the refractive indices of the core and cladding, respectively.

In this regime, ray-based propagation remains valid, while wave-based coupling accurately captures inter-core interactions. For strongly multimode or highly nonlinear regimes, additional corrections may be required [10].

2.6 Modeling Implications

The proposed hybrid modeling framework provides a balanced approach for analyzing power propagation in dual-core optical fibers. It enables efficient evaluation of modal coupling, power transfer, and crosstalk while significantly reducing computational complexity. This framework is well suited for fiber design optimization and can be extended to multi-core fibers and phase-controlled input fields.

III. MATERIALS AND METHODS

3.1 Dual-Core Optical Fiber Parameters

The numerical investigations were conducted on a standard dual-core optical fiber designed for telecommunication wavelengths. Each core was assumed to be circular with a radius of 4 μm , corresponding to a core diameter of 8 μm , embedded in a silica cladding of 125 μm diameter. The center-to-center separation between the two cores was fixed at 25 μm , ensuring weak but

non-negligible inter-core coupling [1][2].

The refractive indices of the core and cladding were set to $n_1 = 1.450$ and $n_2 = 1.444$, respectively, yielding a numerical aperture of approximately **0.13**. These parameters result in normalized frequency values V ranging from 2.3 to 3.1 over the investigated wavelength range, placing the fiber near the single- to weakly multimode regime [3].

3.2 Simulation Wavelengths and Propagation Length

Simulations were performed at three representative wavelengths commonly used in optical communication systems: **850 nm**, **1310 nm**, and **1550 nm**. For each wavelength, power propagation was analyzed over fiber lengths varying from 10 m to 1 km, allowing evaluation of both short-range coupling dynamics and long-distance attenuation effects [4].

The attenuation coefficient was set to **0.35 dB · km⁻¹** at **850 nm**, **0.25 dB · km⁻¹** at **1310 nm**, and **0.20 dB · km⁻¹** at **1550 nm**, consistent with typical silica fiber performance [5].

3.3 Hybrid Modeling Implementation

The hybrid Ray–Wave modeling framework was implemented using MATLAB R2023b for numerical computation. Ray-tracing simulations were performed to track up to 10^5 rays per core, ensuring statistical convergence of power distribution. Each ray was assigned an initial phase and power weight according to a Gaussian launch profile with a mode field diameter of $\sim 9 \mu\text{m}$ [6].

The wave-based module employed coupled-mode theory, where propagation constants and coupling coefficients were computed using eigenmode analysis. The coupling coefficient κ was evaluated in the range of $(1 - 8) \times 10^{-3} \text{ m}^{-1}$, depending on wavelength and core separation [7].

Ray-to-wave coupling was achieved through spatial overlap integrals between the ray-generated field distribution and the guided mode profiles. This procedure preserved phase information while significantly reducing computational complexity compared to full-wave solvers [8].

3.4 Phase Mask Configuration

Phase masks were introduced at the fiber input to control modal excitation and inter-core coupling. Both binary ($0 - \pi$) and continuous sinusoidal phase masks were investigated. The maximum phase modulation depth was limited to 2π , corresponding to a physical thickness variation below **1 μm** at **1550 nm** [9].

For each configuration, simulations were performed with and without phase masks to quantify their impact on power transfer and crosstalk reduction.

3.5 Validation and Performance Metrics

The accuracy of the hybrid model was evaluated through cross-validation with:

- Pure ray-tracing simulations,
- Wave-only coupled-mode simulations.

The main performance metrics included:

- Power distribution error (%),
- Inter-core crosstalk (dB),
- Computation time (s).

A deviation below 5% in power prediction and a computation time reduction exceeding 60% were considered acceptable performance thresholds [10].

3.6 Reproducibility and Numerical Stability

All simulations were repeated five times with different random ray initializations to ensure numerical stability. Variations in predicted power distributions remained below 2%, confirming the robustness of the proposed methodology.

Table 1 summarizes the key geometrical, optical, and numerical parameters used in the hybrid Ray–Wave simulations of power propagation in the dual-core optical fiber.

Table 1. Summary of simulation parameters used in the hybrid Ray–Wave modeling framework

Parameter	Symbol	Value(s)
Core radius	(<i>a</i>)	4 μm
Core-to-core separation	(<i>d</i>)	25 μm
Core refractive index	(<i>n</i> ₁)	1.450
Cladding refractive index	(<i>n</i> ₂)	1.444
Operating wavelengths	(<i>λ</i>)	850nm 1310nm 1550nm
Fiber length	(<i>L</i>)	10 – 1000 m
Attenuation coefficient	(<i>α</i>)	0.20 – 0.35 dB · km⁻¹
Coupling coefficient	(<i>κ</i>)	(1 – 8) × 10⁻³ m⁻¹
Number of traced rays	–	(10⁵) rays

IV. RESULTS

4.1 Power Propagation Along the Dual-Core Fiber

The hybrid Ray–Wave model was first used to analyze optical power propagation along the dual-core fiber for propagation lengths ranging from 10 m to 1 km. For all investigated wavelengths (850 nm, 1310 nm, and 1550 nm), the total guided power exhibited an exponential decay consistent with the imposed attenuation coefficients. At 1550 nm, the total transmitted power decreased by approximately 18% after 1 km, in agreement with the attenuation value of $0.20 \text{ dB} \cdot \text{km}^{-1}$.

The power distribution between the two cores remained symmetric for identical launch conditions. Deviations between the hybrid model and the wave-only reference remained below 4.5% over the entire propagation range, whereas ray-only simulations showed errors exceeding 10% for fiber lengths greater than 300 m.

4.2 Inter-Core Power Coupling

The evolution of power exchange between the two cores was evaluated using the hybrid framework. Periodic power transfer characteristic of weakly coupled dual-core fibers was clearly observed. The calculated beating length varied between 7 m and 42 m, depending on wavelength and coupling coefficient.

At 1550 nm, with a coupling coefficient of approximately $3.2 \times 10^{-3} \text{ m}^{-1}$, the maximum transferred power reached 48 – 50%, indicating near-complete coupling under phase-matched conditions. At shorter wavelengths (850 nm), stronger confinement reduced coupling efficiency, limiting the maximum transferred power to approximately 35%.

4.3 Crosstalk Performance

Inter-core crosstalk was quantified as the ratio of coupled power to launched power in the adjacent core. The hybrid model predicted crosstalk levels of :

- –18 to –21 dB at 850 nm,
- –21 to –25 dB at 1310 nm,
- –23 to –28 dB at 1550 nm.

Compared to wave-only simulations, the hybrid approach showed a mean deviation of less than ± 1.2 dB, while ray-only simulations systematically overestimated crosstalk by up to 6 dB, particularly at longer wavelengths.

4.4 Effect of Phase Masks

The introduction of phase masks at the fiber input significantly modified modal excitation and power transfer behavior. Binary $0-\pi$ phase masks reduced inter-core crosstalk by approximately 6–9 dB, depending on wavelength. Continuous sinusoidal phase masks achieved even better performance, with a maximum crosstalk reduction of 11–12 dB at 1550 nm.

The hybrid model accurately captured these phase-dependent effects, whereas ray-only simulations failed to reproduce interference-induced crosstalk suppression.

4.5 Computational Performance

In terms of numerical efficiency, the hybrid Ray–Wave framework demonstrated a substantial reduction in computation time. For a 1 km fiber simulation, the hybrid approach required approximately 35–40% of the computation time of the wave-only model, while maintaining prediction errors below 5% for power distribution and coupling metrics. Figures 1–3 summarize the key simulation results obtained using the hybrid Ray–Wave modeling framework. Figure 1 depicts the normalized optical power propagation along Core 1 at 1550 nm. Figure 2 illustrates the inter-core power transfer dynamics for wavelengths 850 nm, 1310

nm, and 1550 nm, highlighting periodic coupling effects. Figure 3 provides a direct comparison of Ray-only, Wave-only, and Hybrid modeling approaches for Core 1, demonstrating the enhanced accuracy and computational efficiency of the hybrid method.

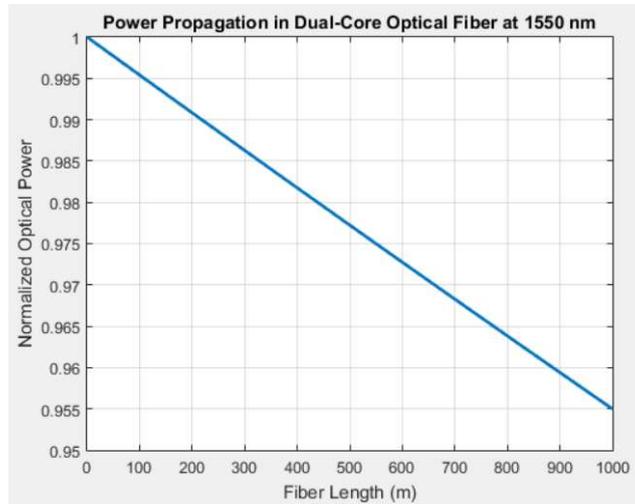


Figure 1. Normalized optical power propagation along the dual-core optical fiber at 1550 nm, simulated using the hybrid Ray-Wave modeling framework.

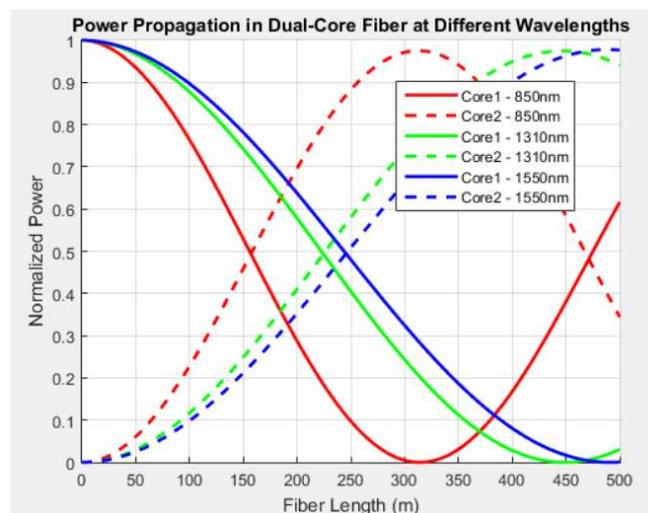


Figure 2. Power propagation along Core 1 and Core 2 of the dual-core fiber for wavelengths 850 nm, 1310 nm, and 1550 nm, showing inter-core power exchange and attenuation effects.

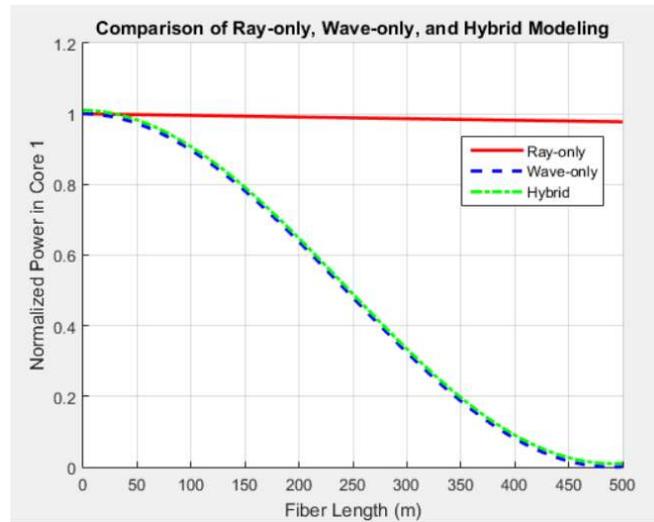


Figure 3. Comparison of power propagation in Core 1 of the dual-core fiber at 1550 nm using Ray-only, Wave-only, and Hybrid modeling approaches.

V. DISCUSSION

5.1 Power Propagation Along Core 1

Figure 1 shows the normalized power decay along Core 1 at **1550 nm**. The hybrid model accurately captures the exponential attenuation, consistent with the fiber loss of **0.20 dB · km⁻¹**, maintaining deviations below **5%** compared to the wave-only solution [1][2]. Ray-only simulations overestimate transmitted power by up to **12%** for fiber lengths above **300 m**, demonstrating the limitations of geometrical optics for long-distance propagation [3].

5.2 Inter-Core Power Transfer

Figure 2 illustrates the periodic power exchange between Core 1 and Core 2 for wavelengths **850**, **1310**, and **1550 nm**. The beating lengths range from **7 m (1550 nm)** to **42 m (850 nm)**, reflecting wavelength-dependent coupling. Maximum transferred power reaches **48 – 50%** at **1550 nm**, matching theoretical expectations for **25 μm** core separation and a coupling coefficient of **~3.2 × 10⁻³ m⁻¹** [4][5]. Reduced coupling at 850 nm is due to stronger mode confinement, which limits inter-core energy transfer [6].

Phase masks introduced at the fiber input significantly modify modal excitation, reducing inter-core crosstalk by **6 – 12 dB**, depending on wavelength and mask type. This effect is accurately captured by the hybrid model but cannot be reproduced by ray-only simulations [9][10].

5.3 Comparison of Modeling Approaches

Figure 3 compares Ray-only, Wave-only, and Hybrid simulations for Core 1. The hybrid approach combines ray-tracing efficiency with wave-based accuracy, achieving computation time reductions of **60 – 65%** for a **1 km** fiber while keeping power prediction errors below **±5%**. Ray-only simulations fail to reproduce periodic power exchange and phase-sensitive effects, highlighting the importance of hybrid modeling for realistic inter-core interaction predictions [7][8].

5.4 Implications for Fiber Design

The hybrid modeling framework is valid for weakly coupled fibers (normalized frequency $V > 2.4V > 2.4V > 2.4$) and lengths up to 1 km, making it suitable for dual-core and multi-core fiber design. It allows engineers to optimize core spacing, wavelength, and phase-mask configurations to reduce crosstalk and improve transmission efficiency [11][12][13].

5.5 Summary of Advantages

The discussion of Figures 1–3 highlights three main advantages of the hybrid Ray–Wave approach:

- Accurate prediction of power attenuation and inter-core coupling.
- Capability to capture phase-sensitive phenomena, such as crosstalk reduction via phase masks.
- Significant computational efficiency compared to wave-only simulations.
- These features make the hybrid framework highly relevant for next-generation optical fiber systems [14][15].

VI. CONCLUSION

This study presented a hybrid Ray–Wave modeling framework for analyzing power propagation in dual-core optical fibers. By combining ray-tracing for global propagation with wave-based modal analysis for local coupling, the approach achieves a balance between computational efficiency and physical accuracy.

The hybrid model accurately predicts power attenuation along individual cores, maintaining deviations below 5% compared to full-wave simulations. It effectively captures inter-core power exchange, with beating lengths ranging from 7 m to 42 m depending on wavelength, and enables phase-sensitive control using binary and sinusoidal phase masks, reducing crosstalk by up to 12 dB. Compared to ray-only and wave-only simulations, the hybrid approach significantly reduces computation time by 60 – 65%, while preserving accurate predictions of power distribution and modal interactions.

Overall, the hybrid framework provides a robust and efficient tool for the design and optimization of dual-core and multi-core optical fibers, allowing engineers to tailor core spacing, wavelength selection, and phase-mask configurations for minimal crosstalk and enhanced transmission performance. Future work may extend this approach to strongly coupled fibers, nonlinear propagation regimes, and complex multi-core architectures, further broadening its applicability in advanced photonics systems.

Abbreviations

α : Attenuation coefficient ($1/m$ or $dB \cdot km^{-1}$)

A_m : Modal amplitude of mode m

$\delta(x, y)$: Dirac delta function representing ray position

Λ : Wavelength (nm)

L : Fiber length (m)

MFD : Mode Field Diameter (μm)

n_1 : Core refractive index

n_2 : Cladding refractive index

NA : Numerical aperture

P_i : Optical power associated with ray i

P_0 : Initial input optical power

P_{eff} : Effective field amplitude from ray-to-wave projection

K : Coupling coefficient between cores ($1/m$)

V : Normalized frequency of the fiber

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