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Classification of Tutoring Teachers Uses the Concept of The Correlation Coefficient

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Abstract— Picture Fuzzy Set is a development of the intuitionistic fuzzy set concept. Picture Fuzzy Set is modeled in a situation when facing human opinion which involves more answers: yes, abstention or neutral, no and rejection. Therefore, to deal with situations like this, the concept of the proposed correlation coefficient is used to calculate the degree of correlation between picture fuzzy set that aim to group different objects. The correlation concept used in picture fuzzy set uses two formulas. And in deciding on a problem, we use the grouping method for picture fuzzy sets introduced by Xu et al [11]. There are four forms of classification.

Keywords—Concept, Correlation coefficient, Picture fuzzy set, Classification.

I. INTRODUCTION

Intuitionistic fuzzy set (IFS) as an extension of the fuzzy set (FS) was first introduced by L. A. Zadeh in 1963 [12]. Since initially introduced, intuitionistic fuzzy set theory has been widely used in various fields, such as decision-making, logic programming, pattern recognition, medical diagnosis, and cluster analysis. The concept of this intuitionistic fuzzy set was developed in a concept related to the concept of correlation coefficient inspired from the concept of the correlation coefficient in the field of statistics. Correlation coefficient is a value used to measure the strength or not the relationship between the two variables [10]. The correlation coefficient is often used in statistics, including in data analysis, classification, pattern recognition, and decision-making. In statistical data, sometimes the data obtained relates to problems in life and uncertain daily value. In overcoming this problem, many researchers extend the notion of statistical correlation to fuzzy correlation. Gerstenkorn and Manko [4] introduced the correlation coefficient on intuitionistic fuzzy sets in probability space. Mitchell [8] proposed correlation coefficients from intuitionistic fuzzy sets by interpreting intuitionistic fuzzy sets as fuzzy sets. Hung and Wu [6] developed a method to calculate the correlation coefficient intuitionistic fuzzy set with the "centroid" method.

In a decision-making problem, anyone can be solved with intuitionistic fuzzy set, but some cannot be solved by an intuitionistic fuzzy set. Therefore, in the year 2013, Cuong [2,3] introduced a picture fuzzy set (PFS) concept which is a development of intuitionistic fuzzy set. Picture fuzzy set modeled in situations when confronted by human opinion involve more answers: yes, abstention or neutral, no, and rejection. Therefore, to encounter such a situation, then used the concept of the proposed correlation coefficient to calculate the degree of correlation between the picture fuzzy set that are aimed at group different objects.

This research aims to apply the concept of the correlation coefficient in classification of tutoring teachers who have a strong relationship based on *the confidence level* λ chosen.

II. FUZZY SET, INTUITIONISTIC FUZZY SET, PICTURE FUZZY SET, CAUCHY-SCHWARZ EQUALITY, CORRELATION COEFFICIENT

In this chapter, we review the theories that will be used in discussion of the correlation coefficient on the picture fuzzy set.

Definition 2.1. [12] Let X be a set of infinite universes. A fuzzy set G over X can be defined as

$$G = \{(x; \mu_G(x)) | x \in X\},$$

where $\mu_G: X \to [0, 1]$ is called the membership function, and $\mu_G(x)$ called degree of membership of $x \in X$ in the fuzzy set G.

Definition 2.2. [1] Let X be a set of infinite universes, an intuitionistic fuzzy set H over X can be defined as

$$H = \{(x; \mu_H(x); \nu_H(x)) | x \in X\},\$$

where $\mu_H: X \to [0, 1]$ and $\nu_H: X \to [0, 1]$ successively stated membership function and non-membership function on intuitionistic fuzzy set H, $\mu_H(x)$ is called the degree of membership of $x \in X$ in the intuitionistic fuzzy set H, and $\nu_H(x)$ is called the degree of non-membership of $x \in X$ in the intuitionistic fuzzy set H. Next, for every $x \in X$ must satisfy

$$0 \le \mu_H(x) + \nu_H(x) \le 1$$

and $\pi_H(x) = 1 - (\mu_H(x) + \nu_H(x))$ is called the degree of hesitation of $x \in X$ in the intuitionistic fuzzy set H, with $\pi_H(x) : X \to [0,1]$. It means $\pi_H(x)$ expresses ignorance about whether x has the degree of membership or degree of non-membership on fuzzy set H.

Definition 2.3. [2,3] Let X be a set of infinite universes. A picture fuzzy set over X can be defined a

$$A = \{(x; \ \mu_A(x); \ \eta_A(x); \ \nu_A(x)) | x \in X\},\$$

where $\mu_A(x)$ is called the degree of positive membership of x on A, $\eta_H(x)$ is called the degree of negative membership of x on A, and $\nu_H(x)$ is called the degree of neutral membership of x on A. In the case $\mu_A, \eta_A, \nu_A : X \to [0,1]$ are functions. Next $\mu_A(x), \eta_A(x)$, and $\nu_A(x)$ must satisfy

$$\mu_A(x) + \eta_A(x) + \nu_A(x) \le 1$$

and $\rho_A(x) = 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$ is called the degree of refusal membership of x on A. It means $\rho_A(x)$ expresses ignorance about whether x has the degree of positive membership, the degree of negative membership, or the degree of neutral membership on picture fuzzy set A.

Definition 2.4. [2,3] Let PFS(X) be a set of all picture fuzzy sets over the universe set X. For every two-picture fuzzy set A and picture fuzzy set B, the union, intersection, and complement operations are defined as follow:

- i. $A \subseteq B$, if and only if $(\forall x \in X; \mu_A(x) \le \mu_B(x); \eta_A(x) \le \eta_B(x); \text{ and } \nu_A(x) \ge \nu_A(x))$,
- ii. A = B, if and only if $(A \subseteq B \text{ and } B \subseteq A)$,
- iii. $A \cup B = \{(x; \max(\mu_A(x); \mu_B(x)); \min(\eta_A(x); \eta_B(x)); \min(\nu_A(x); \nu_A(x)) | x \in X)\},$
- iv. $A \cap B = \{(x; \min(\mu_A(x); \mu_B(x)); \min(\eta_A(x); \eta_B(x)); \max(\nu_A(x); \nu_A(x)) | x \in X)\},\$
- v. $CoA = \bar{A} = \{(x; \nu_A(x); \eta_A(x); \mu_A(x)) | x \in X\}.$

Theorem 2.1. [7] Let $x_1, x_2, ..., x_n, y_1, y_2, ..., y_n$ be real numbers, then $(x_1y_1 + x_2y_2 + ... + x_ny_n)^2 \le (x_1^2 + x_2^2 + ... + x_n^2)$ $(y_1^2 + y_2^2 + ... + y_n^2)$ or can be written as follows:

$$\left(\sum_{i=1}^{n} x_{i} y_{i}\right)^{2} \leq \left(\sum_{i=1}^{n} x_{i}^{2}\right) \left(\sum_{i=1}^{n} y_{i}^{2}\right).$$

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Definition 2.5.[9] Let $X = \{x_1, x_2, ..., x_n\}$ be a set of infinite universes. An informational energy over A, by $A = \{(x_i; \mu_A(x_i); \eta_A(x_i); \nu_A(x_i)) | x_i \in X, i = 1, 2, ..., n\}$, can be defined as

$$E_p(A) = \sum_{i=1}^n (\mu_A^2(x_i) + \eta_A^2(x_i) + \nu_A^2(x_i) + \rho_A^2(x_i)).$$

Definition 2.6.[9] Let $X = \{x_1, x_2, ..., x_n\}$ be a set of infinite universes. A correlation two picture fuzzy set A and picture fuzzy B over X, by $A = \{(x_i; \mu_A(x_i); \eta_A(x_i); \nu_A(x_i)) | x_i \in X\}$, and $B = \{(x_i; \mu_B(x_i); \eta_B(x_i); \nu_B(x_i)) | x_i \in X\}$, can be defined as

$$C_p(A,B) = \sum_{i=1}^n (\mu_A(x_i)\mu_B(x_i) + \eta_A(x_i)\eta_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \rho_A(x_i)\rho_B(x_i)).$$

Proposition 2.1.[2] A correlation between picture fuzzy set A and picture fuzzy set B over X, $C_p(A, B)$, satisfies the following properties:

- i. $C_p(A, A) = E_p(A)$,
- ii. $C_p(A,B) = C_p(B,A)$.

Definition 2.7.[9] Let $X = \{x_1, x_2, ..., x_n\}$ be a set of infinite universes. A correlation coefficient between two picture fuzzy set A and picture fuzzy set B over X, by $A = \{(x_i; \mu_A(x_i); \eta_A(x_i); \nu_A(x_i)) | x_i \in X\}$, and $B = \{(x_i; \mu_B(x_i); \eta_B(x_i); \nu_B(x_i)) | x_i \in X\}$, can be defined as

$$K_p(A,B) = \frac{c_{p(A,B)}}{\left[E_{p(A)} \cdot E_{p(B)}\right]^{\frac{1}{2}}}.$$

Theorem 2.2.[9] Let given two picture fuzzy set A and picture fuzzy set B over X, then $K_p(A, B)$ satisfies the following conditions:

- i. $K_p(A,B) = K_p(B,A)$,
- ii. $0 \le K_p(A, B) \le 1$, and
- iii. If A = B, then $K_p(A, B) = 1$.

In the decision-making process, for example in classification several picture fuzzy sets based on certain criteria, the objects in picture fuzzy set can also considered to have unequal weight. Which object considered usually have different interests, so can be assigned different weights. Let w be a weight vector where $w = (w_1, w_2, ..., w_n)$ with $w_i \in [0,1]$, and $\sum_{i=1}^n w_i = 1$. Then expansion of the formula Definition 2.7 can be defined as follows:

$$K_{pw}(A,B) = \frac{C_{pw}(A,B)}{\left[E_{pw}(A) \cdot E_{pw}(B)\right]^{\frac{1}{2}}}(*)$$

 $with \qquad C_{pw}(A,B) = \sum_{i=1}^{n} w_i (\mu_A(x_i)\mu_B(x_i) + \eta_A(x_i) \, \eta_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \rho_A(x_i)\rho_B(x_i)). \\ E_{pw}(A) = \sum_{i=1}^{n} w_i (\mu^2_A(x_i) + \mu^2_A(x_i) + \mu^2_A(x_i)$

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Theorem 2.3.[9] Let $w = (w_1, w_2, ..., w_n)^T$ be a weight vector of x_i (i = 1, 2, ..., n) with $w_i \in [0, 1]$, and $\sum_{i=1}^n w_i = 1$, then the correlation coefficient between two picture fuzzy set A and picture fuzzy set B over X $K_p(A, B)$ satisfies the following conditions:

- i. $K_{pw}(A,B) = K_{pw}(B,A)$,
- ii. $0 \le K_{pw}(A, B) \le 1$, and
- iii. If A = B, then $K_{pw}(A, B) = 1$.

Definition 2.8.[9] Let $A_j = (j = 1, 2, ..., m)$ be m picture fuzzy sets, and $C = (K_{ij})_{mxm}$ is called the correlation matrix, with $K_{ij} = (A_i, A_j)$ is the correlation coefficient between two picture fuzzy set A_i and picture fuzzy set A_j which satisfies the following conditions:

- i. $K_{ij} = K_{ii}$, for all i, j = 1, 2, ..., m,
- ii. $0 \le K_{ij} \le 1$, for all i, j = 1, 2, ..., m, and
- iii. $K_{ij} = 1$, for all i, j = 1, 2, ..., m.

Definition 2.9.[11] Let $C = (K_{ij})_{mxm}$ be a correlation matrix. If matrix C is operated with C itself which is symbolized by $C^2 = C \circ C = (\overline{K_{ij}})_{mxm}$, where

$$\overline{K_{ij}} = \max \left\{ \min_{k} \{K_{ik}, K_{kj}\} \right\},\,$$

then C^2 is called a composition matrix in C.

Theorem 2.4.[11] Let C be a correlation matrix, then the composition matrix $C^2 = C \circ C = (\overline{K_{ij}})_{mxm}$ is called the correlation matrix.

Definition 2.10.[11] Let $C = (K_{ij})_{mxm}$ be a correlation matrix. C is called the equivalent correlation matrix, if $C^2 \subseteq C$ means

$$max \left\{ min_{k} \{K_{ik}, K_{kj}\} \right\} \le K_{ij}, i, j = 1, 2, ..., m.$$

Theorem 2.5.[11] Let $C = (K_{ij})_{mxm}$ be a correlation matrix. For example, the symbol $C^{2^k} \to C^{2^{(k+1)}}$ which means that

$$C^{2^{(k+1)}} = C^{2^k} \circ C^{2^k}.$$

For every positive integer k, C^{2^k} is a correlation matrix. Next for the sequence of composition

$$C \to C^2 \to C^4 \to \cdots \to C^{2^k} \to \cdots$$

there is a positive integer k such that

$$C^{2^k} = C^{2^{(k+1)}}$$

and

$$C^{2^k}$$

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is also an equivalent correlation matrix.

Definition 2.11.[11] Let $C = (K_{ij})_{mxm}$ be an equivalent correlation matrix, then $C_{\lambda} = ({}_{\lambda}K_{ij})_{mxm}$ is called the λ -cutting matrix of C, where

$$_{\lambda}K_{ij} = \begin{cases} 0, if \ K_{ij} < \lambda \ (i, j = 1, 2, ..., m), \\ 1, if \ K_{ij} \ge \lambda \ (i, j = 1, 2, ..., m). \end{cases}$$

and λ is called the confidence level with $\lambda \in [0,1]$.

Next will be given the decision-making problem algorithm as the application of the correlation coefficient concept.

Algorithm:

- 1. Let $A = \{A_1, A_2, ..., A_m\}$ be a set of picture fuzzy sets over $X = \{x_1, x_2, ..., x_n\}$. Calculate the correlation coefficient of two picture fuzzy sets, and then construct a correlation matrix $C = (K_{ij})_{mxm}$, where $K_{ij} = K = (A_i, A_j)$.
- 2. Check whether $C = (K_{ij})_{mxm}$ is an equivalent correlation matrix. If C is not an equivalent correlation matrix, then do the composite matrix composition until obtained $C^{2^k} = C^{2^{(k+1)}}$, and C^{2^k} is an equivalent correlation matrix.
- 3. For a confidence level $\lambda \in [0,1]$, a λ -cutting matrix is constructed where $C_{\lambda} = (\ _{\lambda}K_{ij})_{mxm}$ by using Definition 2.11. to classify picture fuzzy sets A_j ($j=1,2,\ldots,m$). If the entries in the i-th row (or column) in matrix C_{λ} is equal to the corresponding entries in the row (or column) j, then the picture fuzzy set A_i and picture fuzzy set A_j have "same type". The same type means that picture fuzzy set A_i related to the picture fuzzy set A_j . A picture fuzzy set A_i and picture fuzzy set A_i said to be related or not related as follows:
 - i. The picture fuzzy set A_i is related to the picture fuzzy set A_j , if $K_{ij} \ge \lambda$, which is denoted by 1,
 - ii. The picture fuzzy set A_i is not related to the picture fuzzy set A_i , if $K_{ij} < \lambda$, which is denoted by 0.

Through this principle, all m picture fuzzy sets can be classified $A_i(j = 1, 2, ..., m)$.

III. RESULT AND DISCUSSION

This section will provide a case that can be represented as the concept of correlation coefficient about the construction of the decision-making algorithm correlation coefficient concept.

In this case, the authors do not explain in detail the technical and accuracy of obtaining data because this requires separate and in-depth research involving experts in the field under study. Therefore, the data in this case is only illustrative. The next case to be studied is the case of classifying the tutoring teachers.

Case. A tutoring institute called The Pancaran Ilmu's Bimble Radiance wants to do a performance appraisal of four teachers. For example, $A = \{A_1, A_2, A_3, A_4\}$ is a set of teachers in Pancaran Ilmu's Bimble. Four teachers will be assessed based on three different criteria denoted by $X = \{x_1, x_2, x_3\}$, where $x_1 =$ "pedagogical", $x_2 =$ "personality", $x_3 =$ "professional". In giving an assessment the leader of institute consider the weight based on the importance of each criterion which is given with weight w = (0.5; 0.3; 0.2) on each criterion assessed in decision making. Next for each teacher, will be awarded each degree of positive membership, degree of neutral membership, and degree of negative membership to each criterion. Following are the results of each teacher's assessment shown in table

PFS	x_1			x_2			x_3		
	$\mu_{A_1}(x_1)$	$\eta_{A_1}(x_1)$	$v_{A_1}(x_1)$	$\mu_{A_1}(x_2)$	$\eta_{A_1}(x_2)$	$v_{A_1}(x_2)$	$\mu_{A_1}(x_3)$	$\eta_{A_1}(x_3)$	$\nu_{A_1}(x_3)$
A_1	0.8	0.1	0.1	0.8	0.2	0.0	0.9	0.0	0.1
A_2	0.6	0.2	0.1	0.9	0.1	0.0	0.5	0.1	0.2

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A_3	0.6	0.1	0.2	0.7	0.1	0.1	0.6	0.2	0.1
A_4	0.5	0.1	0.3	0.8	0.1	0.1	0.7	0.1	0.1

Problems in the decision-making process carried out to group four teachers into groups which has a strong relationship based on the level of confidence λ which chosen. In the decision-making process used steps algorithm to classify all teachers $A_j(j=1,2,3,4)$. The following is an algorithm for solving problems in the process decision-making given in the illustrative example.

Step 1. Using equation (*), compute the correlation coefficients of picture fuzzy set A_j (j = 1,2,3,4) and then construct the correlation matric C.

$$C = \begin{bmatrix} 1 & 0.9442 & 0.9617 & 0.9413 \\ 0.9442 & 1 & 0.9658 & 0.9531 \\ 0.9617 & 0.9658 & 1 & 0.9791 \\ 0.9413 & 0.9531 & 0.9791 & 1 \end{bmatrix}$$

Step 2. Construct equivalent correlation matrix

$$C^2 = C \circ C = \begin{bmatrix} 1 & 0.9617 & 0.9617 & 0.9617 \\ 0.9617 & 1 & 0.9658 & 0.9658 \\ 0.9617 & 0.9658 & 1 & 0.9791 \\ 0.9617 & 0.9658 & 0.9791 & 1 \end{bmatrix}$$

Because $C^2 \nsubseteq C$, then C is not equivalent correlation matrix. So, it counts return the correlation matrix entries until the results are obtained $C^{2^k} = C^{2^{(k+1)}}$. Then calculate C^4 as follows

$$C^4 = C^2 \circ C^2 = \begin{bmatrix} 1 & 0.9617 & 0.9617 & 0.9617 \\ 0.9617 & 1 & 0.9658 & 0.9658 \\ 0.9617 & 0.9658 & 1 & 0.9791 \\ 0.9617 & 0.9658 & 0.9791 & 1 \end{bmatrix}$$

Because $C^4 = C^2$, this shows C^2 is the equivalent correlation matrix.

Step 3. Classify four teachers A_j (j = 1,2,3,4) based on the confidence level λ chosen with using Definition 2.11 of the equivalent correlation matrix. Here are four forms of classification based on the value of λ chosen for classify four teachers who have "the same type."

1. If $0 \le \lambda \le 0.9617$, then

From the matrix above, it can be concluded that for each $A_i(j = 1,2,3,4)$ can be classified into the same types:

$$\{A_1, A_2, A_3, A_4\}.$$

2. If $0.9617 < \lambda \le 0.9658$, then

$$C^{2}_{\lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

From the matrix above, it can be concluded that for each $A_j(j = 1,2,3,4)$ can be classified into two types:

$${A_1}, {A_2, A_3, A_4}.$$

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3. If $0.9658 < \lambda \le 0.9791$, then

$$C^{2}_{\lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

From the matrix above, it can be concluded that for each $A_i(j = 1,2,3,4)$ can be classified into three types:

$${A_1}, {A_2}, {A_3}, {A_4}..$$

4. If $0.9791 < \lambda \le 1$, then

$$C^{2}_{\lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the matrix above, it can be concluded that for each A_i (i = 1,2,3,4) can be classified into four types:

$${A_1}, {A_2}, {A_3}, {A_4}.$$

IV. CONCLUSION

The application of the concept of the correlation coefficient on the picture fuzzy set in decision making by The Pancaran Ilmu's Bimble Radiance, on the performance appraisal of each A_j (j = 1,2,3,4), was carried out to group the four teachers into groups with strong ties. Based on λ is selected, four forms of classification are obtained.

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