International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

Vol. 39 No. 1 June 2023, pp. 465-466

The Set Multipartite Ramsey Numbers $\mathbf{M_j(P_n, mK_2)}$

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Abstract – For given two any graph H and G, the set multipartite Ramsey number $M_j(G,H)$ is the smallest integer t such that for every factorization of graph $Kt{\times}j := F1 \oplus F2$ so that F_1 contain G as a subgraph or F_2 contains H as a subgraph. In this paper, we determine $M_j(P_n, mK_2)$ with $j = 3, 4, 5$ and $m \geq 2$ where P_n denotes a path for $n = 2, 3$ vertices and mK_2 denotes a matching (stripes) of size m and pairwise disjoint edges.

Keywords – Paths, Set Multipartite Ramsey Numbers, Stripes

I. Introduction

Let $G=(V,E)$ be a graph with the vertex-set $V(G)$ and edge-set $E(G)$. All graphs in this paper are finite and simple. The *minimum degree* and *maximum degree* of G is denoted by $\delta(G)$ and $\Delta(G)$, respectively. The order of the graph G is defined by $|V(G)|$. If $e = uv \in E(G)$ then u is called *adjacent* to v. A graph G is said to be *factorable* into factors $G_1, \cdots, G_n$ if these factors are pairwise edge-disjoint and $\cup_{i=1}^{n} E(G_i) = E(G)$. If G is factored into $G_1, \cdots, G_n$, then $G = G_1 \oplus \cdots \oplus G_n$, which is called a *factorization* of G. A *path* P_n is the graph on $n \geq 2$ vertices with two vertices of degree 1, and n-2 vertices on of degree 2. A m *stripe* of a graph G is defined as a set of m edges without a common vertex.

The notion of set multipartite Ramsey numbers were introduced by Burger and Vuuren [1] in 2004. Let $a, b, c,$ and d be natural numbers with $a, c \geq 2$. The set multipartite Ramsey numbers $M_j(K_{a\times b}, K_{c\times d})$ is the smallest natural number ξ such that an arbitrary colouring of the edges of $K_{\xi \times j}$, using two colours red and blue necessarily forces a red $K_{a\times b}$ or blue $K_{c\times d}$ as a subgraph. In this paper, we generelize this concept by releasing completeness requirement in the forbidden graphs as follows. The definition can be formulated as follows. Given two graphs G_1, G_2, and integer $t \geq 2$, the set multipartite Ramsey numbers $M_j(G_1, G_2) = t$ is the smallest integer such that every factorization of graph $K_{t\times j} := F_1 \oplus F_2$ satisfies the following condition: either F_1 contains G_1 as a subgraph or F_2 contains G_2 as a subgraph of $K_{t\times j}$.

There are only few results on the set multipartite Ramsey numbers $M_j(G,H)$. These are $M_1(K_{2\times 2}, K_{3\times 3}) = 7$ was studied by Chartand and Schuster [3], $M_1(K_{2\times 2}, K_{4\times 1}) = 10$ studied by Chavatal and Harry [2], $M_2(K_{2\times 2}, K_{3\times 1}) = 4$ and $M_2(K_{2\times 2}, K_{4\times 1}) = 7$ studied by Harborth and Mengersen [7,8], $M_1(K_{2\times 2}, K_{5\times 1}) = 14$ studied by Greenwood and Gleason [6], $M_1(K_{2\times 2}, K_{6\times 1}) = 18$ studied by Exoo [5]. In [4], Jayawardene and Samarasekara studied size multiprtite Ramsey numbers for small paths versus stripes. The aim of this paper is determined $M_j(P_n, mK_2)$ with $j = 3,4,5$ for $m \geq 2$. In this note, we prove the following theorem.

II. Set Ramsey numbers related to P_n and mK_2

We will determine the set multipartite Ramsey numbers for path versus stripes as the following theorem.

Theorem 3.1. For positive integer $3 \leq j \leq 5$ and $m \geq 2$, then we have $M_j(P_n, mK_2) = \left\lceil \frac{2m}{j} \right\rceil$.

***Proof*.** Let $s = \left\lceil \frac{2m}{j} \right\rceil$. We will show first that the lower bound of $M_j(P_n, mK_2) \geq s$. Let $F_1 \oplus F_2$ be the any factorization of graph $F = K_{(s-1)\times j}$ such that F_1 contains no P_n for $n = 2,3$ as subgraph. Let $V_i = \{a_{ij}\}$ for $i = 1, 2, 3, \ldots, (s-1)$ and $j = 3,4,5$ be the partite set of F. Since all edges of graph $F = K_{(s-1)\times j}$, then there are not enough vertex to form mK_2 in F_1. Therefore $M_t(P_n, mK_2) \geq s$, for $n = 2,3$.

Next, we will show the upper bound $M_j(P_2, mK_2) \leq s$. Let $G_1 \oplus G_2$ be any the factorization of $G = K_{s\times j}$ such that G_1 contains no P_2 as a subgraph. We will show that G_2 contains mK_2 as a subgraph. Let $V_i = \{a_{ij}\}$ for $i = 1, 2, 3, \ldots, s$ and $j = 3,4,5$ be the partite set of G. Since G_1 contains no P_2 as subgraph, then G_1 is isolated vertex. Hence $|V(G)| = \left(\left\lceil \frac{2m}{j} \right\rceil\right) j$ vertex, then $\frac{sj}{2}$ can form mK_2. As a consequence, G_2 contains mK_2 as a subgraph. Therefore, the set multipartite ramsey numbers $M_j(P_2, mK_2) \leq s$

Next, to show the upper bound $M_j(P_3, mK_2) \leq s$. Let $G_1 \oplus G_2$ be any the factorization of $G = K_{s\times j}$ such that G_1 contains no P_3 as a subgraph. We will show that G_2 contains mK_2 as a subgraph. Let $V_i = \{a_{ij}\}$ for $i = 1, 2, 3, \ldots, s$ and $j = 3,4,5$ be the partite set of G. Since G_1 contains no P_3 as subgraph and $\Delta(G_1) = 1$, then $G_2 = 3(s-1)$. Thus, the complement of G_1 that is G_2 will form mK_2. Hence, G_2 contains mK_2 as a subgraph. Therefore, the set multipartite ramsey numbers $M_j(P_3, mK_2) \leq s$.

III. Conclusions

In this paper, we obtain the set multipartite Ramsey numbers for $M_j(P_n, mK_2)$ for $j = 3,4,5$ and $n = 2,3$ with $m \geq 2$.

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