

Viscous - Radial Oscillations As A Result Of Internal Pressure Of The Cylindrical Shell

G.A. Kulmirzayeva

Senior Teacher Samarkand Institute Of Architecture And Civil Engineering,
Samarkand city, Uzbekistan.



Abstract – In this scientific work be learned the viscous vibrations of the cylindrical shell were studied, while the viscous fluid in the interior was flowing under certain pressure, and the mathematical model of the problem was solved by using the model of integral exchanges and specific issues were solved.

The algebraic equations made using Maple-7 have been removed and the material of the cylindrical shell is argillite and alevrolite, oil is viscous, and graphs are drawn on the results obtained.

Keywords – Shell, cylinder, elastic, fluid, symmetric, pressure, mathematical model, integral substitution, density, argillite, alevrolide, oil, adhesive.

I. INTRODUCTION

In a cylindrical system of coordinates (r, θ, z) the homogeneous and isotropic circular cylindrical elastic layer r_1 with internal and external radii r_2 is considered. In this case, $r_1 = const, r_2 = const, r_2 > r_1$ and the thickness of the layer takes $h = r_2 - r_1$ arbitrary values r_1 and r_2 depending on and. In addition, it is assumed that the cylindrical layer, as a three-dimensional body, strictly obeys the mathematical theory of elasticity and is described by its three-dimensional equations. In addition, it is assumed that the cylindrical layer, as a three-dimensional body, strictly obeys the mathematical theory of elasticity and is described by its three-dimensional equations. It is believed that the inner cavity of the layer is filled with viscous incompressible resting liquid, described by the linearized equations of Navier-Stokes [1]. In the future we will consider the torsional fluctuations of the layer and assume that it is loaded only along the axis Oz [1,2]

Equation of motion of a layer

$$\sigma_{ij,j} = \rho \ddot{U}_i, \quad (i, j = r, \theta, z) \quad x_i \in V \quad (1)$$

Used in the form of Wave equations

$$(\lambda + \mu) \Delta \hat{O} = \rho \frac{\partial^2 \hat{O}}{\partial t^2},$$

$$x_i \in V_1 \quad (2)$$

$$\mu \Delta \vec{\phi} = \rho \frac{\partial^2 \vec{\phi}}{\partial t^2}.$$

For the potentials of longitudinal f and transverse waves $\vec{\phi}$, entered by the formula

$$\vec{U} = \text{grad } \phi + \text{rot}(\vec{e}_3 \phi_1) + \text{rot}(\vec{e}_3 \phi_2), \quad x_i \in V_1 \quad (a)$$

Where is the Laplace operator in the coordinate system (r, θ, z) ;

σ_{ij} , U_i - components of stress tensor and displacement vector;

λ, μ - lamé coefficients; ρ - density V_1 - the volume occupied by the layer.

For viscous incompressible liquid, at its small fluctuations we have the following ratios:

Non-compressibility condition (3)

$$\text{div } \vec{v} = 0, \quad x_i \in V_2 \quad (3)$$

Navier - Stokes equation

$$\frac{\partial \vec{V}}{\partial t} - \nu' \Delta \vec{V} + \frac{1}{\rho_0} \text{grad } p = 0, \quad x_i \in V_2 \quad (4)$$

Navier-Stokes Law (5)

$$P_{ij} = -P \sigma_{ij} + \mu' e_{ij}, \quad x_i \in V_2 \quad (5)$$

Where $\vec{V}$ - the velocity vector of liquid particles;

μ - viscosity coefficient;

$\nu' = \frac{\mu'}{\rho'}$ - kinematic coefficient of viscosity;

ρ_0' - density of the resting liquid;

P - hydrodynamic pressure;

P_{ij} - components of stress tensor in the liquid;

e_{ij} - components of strain velocity Tenzora;

V_2 - volume of space occupied by the liquid (inner cavity of the layer).

The introduction of scalar G and vector functions $\vec{x} = \vec{x}(x_1, x_2)$ according to the formula ,

$$\vec{V} = \frac{\partial}{\partial t} \{grad \ G + zot[\vec{e}_3 x_1 + rot(\vec{e}_3 x_2)]\} \quad (6)$$

Equations (3), (4) are given and the view

$$\Delta G = 0, \left(\frac{\partial}{\partial t} - \nu' \Delta\right) x_1 = 0, \left(\frac{\partial}{\partial t} - \nu' \Delta\right) x_2 = 0 \quad (7) \quad \text{For liquid we put}$$

$$V_r = V_z = 0, \ V_\Theta = V_\Theta(r, z, t), \ p = 0, \quad x_i \in V_2 \quad (8)$$

Then the condition of continuity, taking into account the conditions of incompressibility, should be

$$\frac{\partial \rho'}{\partial t} = 0, \quad x_i \in V_2 \quad (9)$$

According to (6) P and ρ' and through G , x_1 and x_2 conditions (7) and (8) are fulfilled, if in (6) [3,4]

$$G = 0, \quad x_2 = 0, \quad x_1 = x_1(r, z, t) \quad (10)$$

In case (10) of (6) we get to V_Θ represent

$$V_\Theta = - \frac{\partial^2 x_1}{\partial r \partial t}, \quad x_i \in V_2 \quad (11)$$

Where the function X_1 is the solutions of the equation

$$\left[\frac{\partial}{\partial t} - V' \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\right] x_1 = 0, \quad x_i \in V_2 \quad (12)$$

For the layer, accept the following

$$U_r = U_z = 0, \quad U_0 = U_0(r, z, t), \quad x_i \in V_1 \quad (13)$$

Conditions (13) will be performed if put

$$U_\Theta = - \frac{\partial \varphi_1}{\partial r}, \quad x_i \in V_1 \quad (14)$$

Where the function φ_1 based on (2) satisfies the equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right) \varphi_1 = \frac{1}{b^2} \varphi_1, \quad x_i \in V_1 \quad (15)$$

Conditions on the surface of the layer at $r = r_2$ and at the boundary of $r = r_1$ of the environment section are

$$\begin{aligned}\sigma_{r\Theta}(r_2, z, t) &= f_{r\Theta}(z, t), \\ \sigma_{r\Theta}(r_1, z, t) &= p_{r\Theta}(r_1, z, t), \\ \sigma_{r\Theta}(r_1, z, t) &= \frac{\partial}{\partial t} U_{\Theta}(r_1, z, t)\end{aligned}$$

(16) Initial conditions are zero.

Thus, the problem of torsional oscillations of a cylindrical layer with viscous incompressible liquid is given to the solution of equations (12), (15) with boundary (16) and zero conditions [3, 4].

To solve equations (12) and (15) x_1 u φ_1 imagine functions as

$$[x_1, \varphi_1] = \int_0^\infty \frac{\sin kz}{-\cos kz} \} dk \int_{(l)} [x_{10}, \varphi_{10}] e^{pt} dp, \quad (17)$$

The substitution of which in (12) and (15) gives

$$\begin{aligned}\frac{\partial^2 \varphi_{10}}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_{10}}{\partial r} - \alpha^2 \varphi_{10} &= 0, & \alpha^2 &= k^2 + \frac{1}{b^2} p^2, \\ \frac{\partial^2 x_{10}}{\partial r^2} + \frac{1}{r} \frac{\partial x_{10}}{\partial r} - \beta^2 x_{10} &= 0, & \beta^2 &= k^2 + \frac{1}{v'} p^2.\end{aligned} \quad (18)$$

General solutions of equations (18), limited in appearance $r \rightarrow \infty$ u $r = 0$

$$\begin{aligned}\varphi_{10}(r) &= A_1 I_0(\alpha r) + A_2 K_0(\alpha r), \\ x_{10} &= B I_0(\beta r)\end{aligned} \quad (19)$$

The function of external influences is also presented as

$$f_{ro}(z, t) = \int_0^\infty \frac{\sin kz}{-\cos kz} \} dk \int_{(l)} [x_{10}, \varphi_{10}] e^{pt} dp. \quad (20)$$

Expressing the tension through $\sigma_{r\Theta}$ u $p_{r\Theta}$ the entered potentials φ_1 u x_1 , and also presenting them as (20), from the boundary conditions (16),

$$\begin{aligned}\left(\frac{1}{r} \frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2} \right) \varphi_{10} &= \frac{1}{\mu} f_{ro}^{(0)}, & \text{npu } r &= r_2, \\ \left(\frac{1}{r} \frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2} \right) \varphi_{10} &= 2 \frac{\mu'}{\mu} \left[\frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{2} \beta^2 \right] p x_{10}, & \text{npu } r &= r_1\end{aligned}$$

II. FUNDAMENTAL PART

We get to take the desired values of displacement in the points of some intermediate surface of the cylindrical layer, the radius of which is determined by the formula

$$\xi = \left(V - \frac{r_1}{r_2} \right) \quad (21)$$

Having expressed the transformed movement substituting in it general decisions U and U_0 from φ_{10} and x_{10} , movement substituting in it general decisions (19) and using standard expansions of modified functions of Bessel in power series, believing in the resolution $r = \xi$ and proceeding from its general kind we introduce new functions depending on parameters to κ and p , by formulas

$$U_{0,0}^{(0)} = -\frac{1}{2}\alpha^2 \left\{ A_1 - A_2 \left[\ln \frac{\alpha\xi}{2} - \varphi(1) - \frac{1}{2} \right] \right\}, \quad U_{0,0}^{(0)} = \frac{1}{\xi} A_2, \quad (22)$$

By substituting Solutions (19) in the boundary conditions we will get

$$\begin{aligned} \alpha^2 [A_1 I_1(\alpha r_1) + A_2 K_2(\alpha r_2)] &= -\mu^{-2} f_{ro}^{(0)}, \\ \frac{2}{r_1} [A_1 I_1(\alpha r_1) + A_2 K_2(\alpha r_2)] &= -\alpha [A_1 I_0(\alpha r_1) + A_2 K_0(\alpha r_2)] = -\frac{\mu'}{4\mu} \beta^2 p r_1 \alpha^2 [A_1 I_1(\alpha r_1) + A_2 K_2(\alpha r_2)], \end{aligned} \quad (23)$$

Using standard expansions of Bessel functions into power series r_1 and r_2 , by degrees as well as substituting the expressions of constant A_1 and A_2 by formulas (22) and by entering functions by formulas $U_{0,0}$ and $U_{0,1}$ and λ^n from conditions

$$\begin{aligned} [U_{0,0}, U_{0,1}] &= \int_0^\infty \frac{\sin kz}{-\cos kz} dk \int_{(I)} [U_{0,0}^{(0)}, U_{0,1}^{(0)}] e^{pt} dp, \\ \lambda^n(\zeta) &= \int_0^\infty \frac{\sin kz}{-\cos kz} dk \int_{(I)} [\lambda^{2n}(\zeta)] e^{pt} dp. \end{aligned}$$

We get equations (22)

$$\begin{aligned} C_{11} U_{0,0} + C_{12} U_{0,1} &= \mu^{-1} f_{ro}, \\ (C_{12} - R C_{31}) U_{0,0} + (C_{22} - R C_{32}) U_{0,1} &= 0, \end{aligned} \quad (26)$$

Where the operators of C_{ij} have the form

$$C_{1i} = 2 \sum_{n=0}^{\infty} \frac{(r_i/2)^{2n+2}}{n!(n+2)!} \lambda_2^n.$$

Here R is a reaction of viscous incompressible liquid to the vibrations of the shell

$$R = \frac{r_1}{4} \frac{\mu'}{\mu} \frac{\partial}{\partial t} \left(\frac{1}{V} \frac{\partial}{\partial t} - \frac{\partial^2}{\partial z^2} \right) \quad (27)$$

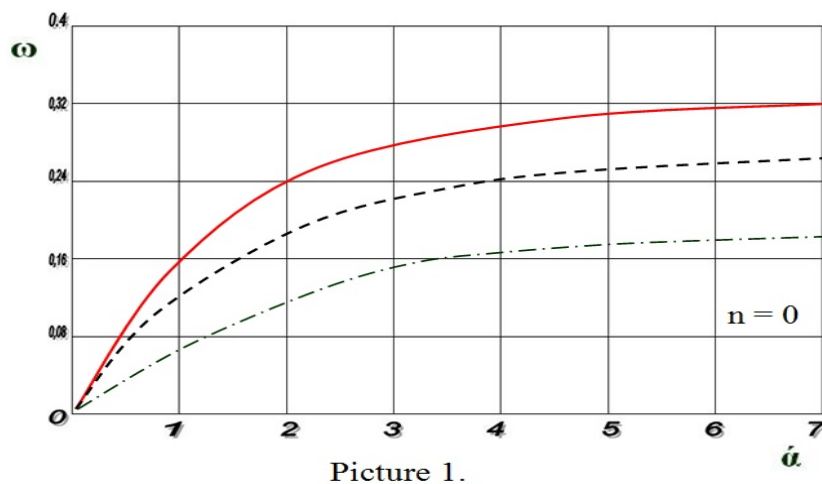
Based on the expression

$$\alpha^2 = k^2 + \frac{1}{b^2} p^2$$

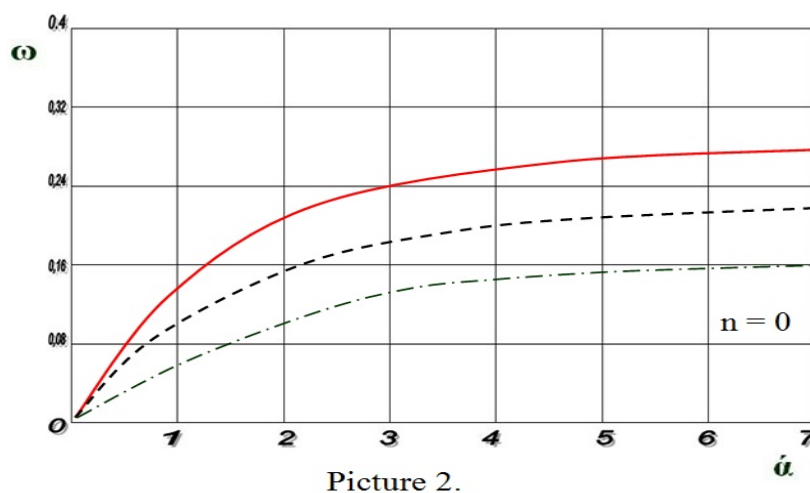
It is easy to conclude that the operators λ^n in the variables (z, t) are equal

$$\lambda^n = \left[\frac{1}{b^2} \left(\frac{\partial^2}{\partial t^2} \right) - \frac{\partial^2}{\partial z^2} \right]^n, \quad n = 1, 2, 3, \dots \quad (28)$$

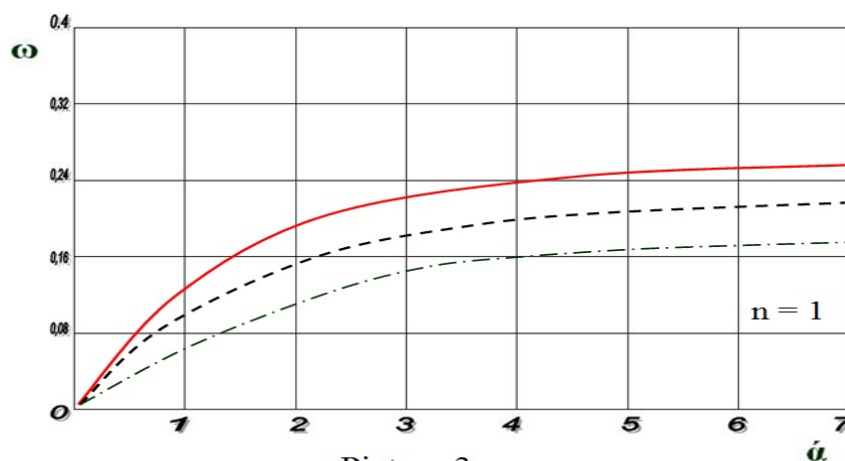
In accordance with (28) equations (27) are the differential equations of infinitely high order relative to the main parts of the circular movement of the points of the intermediate surface of the cylindrical elastic layer with viscous incompressible liquid.



Picture 1. Graph of dependence of oscillatory frequencies from wave numbers at siltstone of a cylindrical layer in absence of a liquid (petroleum), where $h = 0, 1, r_1=1,0, r_2=1,10$.

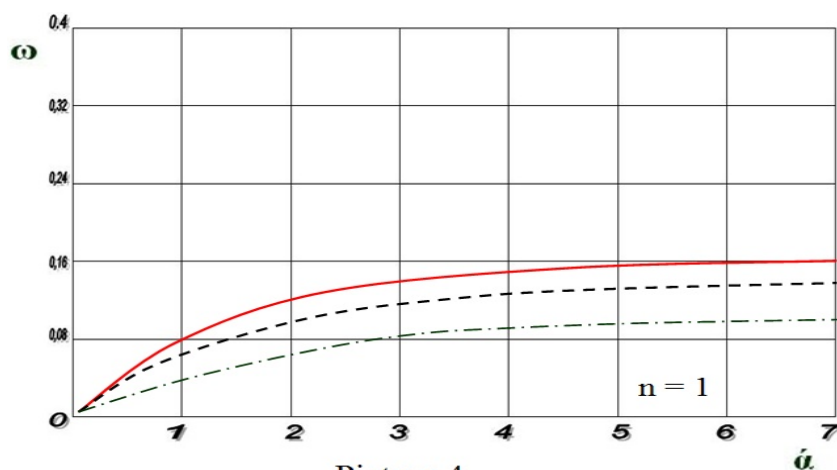


Picture 2. Graph of the dependence of oscillatory frequencies on the wave numbers at the siltstone of the cylindrical layer in the absence of liquid (petroleum), where $h= 0.3, r_1=1,0, r_2=1,30$



Picture 3.

Picture 3. Graph of dependence of oscillatory frequencies from wave numbers at siltstone of a cylindrical layer in filling with petroleum products, where $h = 0, 1$, $r_1=1,0$ $r_2=1,10$.



Picture 4.

Picture 4. Graph of dependence of oscillatory frequencies from cylindrical sloywave numbers at siltstone in filling with petroleum products, where $h = 0, 3$, $r_1=1,0$, $r_2=1,30$

III. CONCLUSION

It is easy to express the movement U_Θ and tension of $\sigma_{r\Theta}, \sigma_{z,\Theta}$ internal sections of the layer and pressure, $P_{r\Theta}$ *чepeз* $U_{\Theta,0}, U_{\Theta,1}$, which by the results of solving equations (27) Allow to determine the stress-strain state of the arbitrary section of the layer and stress on the liquid surface. Note that infinitely high order of equations makes them unsuitable for solving applied problems.

Therefore, considering the fulfilled conditions obtained in earlier works imposed on the frequency of oscillations and the wave number of propagating waves limited to zero ($n = 0$), the first ($n = 1$) and other approximations can get the equations of oscillation suitable To solve engineering problems.

It should be noted that the restrictions are imposed on both the purity and the wave number, which means that the truncated equations do not describe the high-frequency and shortwave processes, and that they are suitable only for low-frequency external influences. In addition, the resulting approximate equations at any approximation are not applicable in the case of concentrated impacts on the system.

However, it does not follow that these equations are applicable to a very narrow class of tasks or do not apply at all, because the functions f_r presented in the form (20), represent a sufficiently broad class and therefore truncated equations have a sufficiently wide Scope of applicability [4] .

Note that infinitely high order of equations makes them unsuitable for solving applied problems. Therefore, considering fulfilled conditions, received in earlier works [4], imposed on frequency of fluctuations wave number of propagating waves and limiting zero ($n = 0$), first ($n = 1$), and other approximations it is possible to get equations of oscillation suitable for solving engineering problems.

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