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On Locating Chromatic Number Of Cubic Graph With Tree Cycle, $C_{n,2n,n}$ *for* 6≤n≤10

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Abstract – **Let** ***G=(V,E)*** **be a connected graph and** ***c*** **be a proper** ***k*****-coloring of** ***G*****. Let** $\prod = \{C_1, C_2, \dots, C_k\}$ **be the partition of** $V(G)$ **induced by coloring** ***c*****. The** ***color code*** $C_{\prod}(v)$ **of a vertex** ***v*** **in** ***G*** **is** $(d(v, C_1), d(v, C_2), \dots, d(v, C_k))$**, where** $d(v, C_i) = min\{d(v, x) \mid x \epsilon C_i\}$ **for** $1 \leq i \leq k$. **If any two distinct vertex** ***u,v*** **in** ***G*** **satisfy that** $C_{\prod}(u) \neq C_{\prod}(v)$, **then** ***c*** **is called** ***a locating k-coloring*** **of G.** ***The locating-chromatic*** **of** ***G*****, denoted by** $\chi_L(G)$**. In this paper, we study the locating coloring of graph cubic** $C_{n,2n,n}$ **for** $6 \leq n \leq 10$**.**

Keywords – ***Locating Chromatic Number, Color Code,Cubic.***

I. Introduction

The locating-chromatic number of a graph G is defined by Chartrand et al. [4] in 2002. This concept is derived from the partition dimension and the graph coloring. The partition dimesion of a graph was firstly studied by Chartrand et al. [5]. Vertex coloring on a graph is the provision of coloring for each vertex on the graph provided that each adjacent vertex must have a different coloring. The minimum number of coloring used for vertex coloring on graph G is called the chromatic number of locating denoted $\chi_L(G)$.

Some of the values of the locating chromatic number that have been obtained include, Chartrand et al. [6] obtained the locating chromatic number of several graph classes, for the path graph P_n with $n > 3$ the locating chromatic number are obtained $\chi_L(P_n) = 3$, for cycle graph obtained $\chi_L(C_n) = 3$, for n odd or $\chi_L(P_n) = 4$, for n even. The locating-chromatic number of firecracker graph by Asmiati et al. [2] in 2012. On locating-chromatic number of graphs with dominat vertex br Welyyanti et al. [7] in 2015. Next, locating-chromatic number of amalgamation of stars by Asmiati et al. [1] in 2011.

Cubic graph are a type of graph. Given a cycle graph with n vertex written as $C_{n,2n,n}$. a cubic graph is denoted by $C_{n_1,n_{2,\dots,}n_{k,}}$ is a graph formed from k cycle that is $C_{n_i}^i$with $i = \{1,2,\dots,k\}$, where $C_{n_i}^i = \{v_{i1}, v_{i2}, \dots, v_{in_i}, v_{i1}\}$ is a graph of cycle i and vertex. In submitted 2020, the chromatic number of the cubic graph locating $C_{n,2n,n}$ for $n = 3,4,5$ and already that the author is interested in studying the problem of the chromatic number of the locating of the cubic graph $C_{n,2n,n}$ for $6 \leq n \leq 10$.

Let us begin to state the following lemma and corollary which are useful to obtain our main result

Lemma 1.1 [4] *Let c be a locating-coloring in a connected graph G. If u and v are distinct vertex of G such that d (u, w) = d (v, w) for all w ∈ V(G)\ {u, v} then c (v) f = c (u). In particular, if u and v are non adjacent vertex of G such that N(u) = N(v), then c (u) f = c (v).*

Corollary 1.2 [4] *If G is a connected graph containing a vertex adjacent to k leaves of G, then* $\chi_L(G) = 3 \geq k + 1$.

A cubic graph denoted by $C_{n_1,n_2,\ldots,n_k}$ is a graph cubic formed from k cycle $C_{n_i}^i$ by $i = 1,2,\ldots,k$, where $C_{n_i}^i = v_{i1}v_{i2},\ldots,v_{in_i v_{i1}}$ is a circular graph in i with n_i vertex. Given a cycle graph with n vertex written as C_n, with $n \geq 3$. Then a cubic graph consisting of three cycle graph namely C_n^1, C_{2n}^2, C_n^3 with $n \geq 3$.

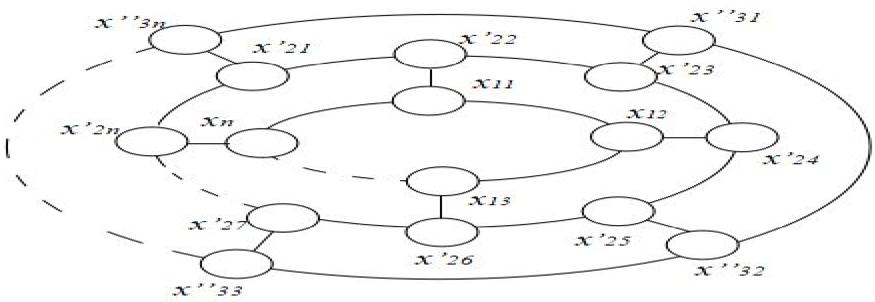

Figure 1. Cubic graph of Cn,2n,n, for n ≥ 3.

II. Main Results

The following theorem will discuss about determined the locating chromatic number of a cubic graph $C_{n,2n,n}$ for $6 \leq n \leq 10$.

Theorem 2.1. *Let be a graph* $C_{n,2n,n}$ *then* $\chi_L(C_{n,2n,n}) = 5$ *for* $6 \leq n \leq 10$.

Proof. Let *c* be the vertex coloring for $6 \leq n \leq 10$ and $\prod = \{S_1, S_2, \ldots, S_k\}$ is a the partition of vertex on the graph where S_i denotes the in i color class for $1 \leq i \leq k$, we will determined the chromatic number of the locating of the cubic graph $C_{n,2n,n}$. For determined the chromatic number of locating, there are several conditions that must be fulfilled. Each adjacent vertex should not be color the same color. Furthermore, the color code of each vertex $C_{n,2n,n}$ to $\prod$ must be different. The following case are.

Case 1. for $n = 6$.

The first shows $\chi_L(C_{6,12,6}) \geq 5$. Note that on the cubic graph $C_{6,12,6}$ there are two C_6 namely C_6^1 and C_6^3, and one C_{12} that is C_{12}^2can be seen in Figure 2.0.1. Because every vertex on $C_{6,12,6}$ has a degree, there will be a dominant vertex with the same color, the color code will be the same, the color is not enough that every vertex in $C_{6,12,6}$ has a different color code will be the same, the color is not enough that every vertex in $C_{6,12,6}$ has a different color code it must be $\chi_L(C_{n,2n,n}) \geq 4$.

Furthermore, it is shown that 4 any color 2 will be a vertex with the same color code at $C_{6,12,6}$. By using the pigeon cage principle, the combination of color ti be used is not sufficient to color the vertex on the graph $C_{n,2n,n}$.

Suppose that the first vertex is colored 1 and the vertices adjacent to the first vertex is colored 2 and 4. Then the second vertices is the vertex other than the first vertex and the adjacent vertex to the first vertex is colored 2 and the vertex adjacent to the second vertex colored with the 1,3,4. Furthermore, the this vertex is a vertex other than the vertex that has been colored previous colored with color 3 and the vertex adjacent the third vertex is colored with color 2 and 4.

Furthermore, the fourth vertex is colored with color 1,2,3 and other vertex colored in any color combination with 4 colors. This the color combination of 4 vertex, there will be two dominant vertex of the same color. It must be $\chi_L(C_{6,12,6}) \geq 5$.

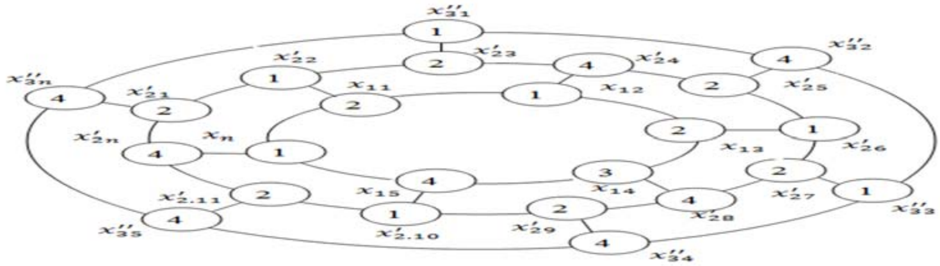

Figure 2.0.1. Coloring with 4 color for C6,12,6.

In Figure 2.0.1 4 −coloring vertex for $C_{6,12,6}$ vertex there are two vertex that have the same color code are $c_\Pi(x_n) = c_\Pi(x_{12}) = (0,1,2,1)$, it must be $\chi_L(C_{6,12,6}) \geq 5$.

Next, we will show $\chi_L(C_{6,12,6}) \leq 5$. Suppose that the construction of the vertex coloring of the cubic graph $C_{6,12,6}$ is defined by $c : V(C_{6,12,6}) \rightarrow \{1,2,3,4,5\}$ such that:

a. $c(x_n) = c(x_{12}) = c(x'_{22}) = c(x'_{26}) = c(x'_{28}) = c(x'_{2.10}) = c(x''_{31}) = c(x'_{33}) = 1$
b. $c(x_{11}) = c(x_{13}) = c(x'_{21}) = c(x'_{23}) = c(x'_{25}) = c(x'_{27}) = c(x'_{29}) = c(x'_{2.11})$=2
c. $c(x_{14}) = 3$
d. $c(x_{15}) = c(x'_{2n}) = c(x'_{24}) = c(x'_{28}) = c(x''_{3n}) = c(x''_{32}) = c(x''_{34}) = 4$
e. $c(x''_{35}) = 5$.

Based on this construction, the following color class are obtained:

$S_1 = \{x_n, x_{12}, x'_{22}, x'_{26}, x'_{28}, x'_{2.10}, x''_{31}, x''_{33}\}$,
$S_2 = \{x_{11}, x_{13}, x'_{21}, x'_{23}, x'_{25}, x'_{27}, x'_{29}, x'_{2.11}\}$,
$S_3 = \{x_{14}\}$,
$S_4 = \{x_{15}, x'_{2n}, x'_{24}, x'_{28}, x''_{3n}, x''_{32}, x''_{34}\}$,
$S_5 = \{x''_{35}\}$.

From the definition of vertex coloring above, it can be seen that each adjacent vertex is colored with a different color.

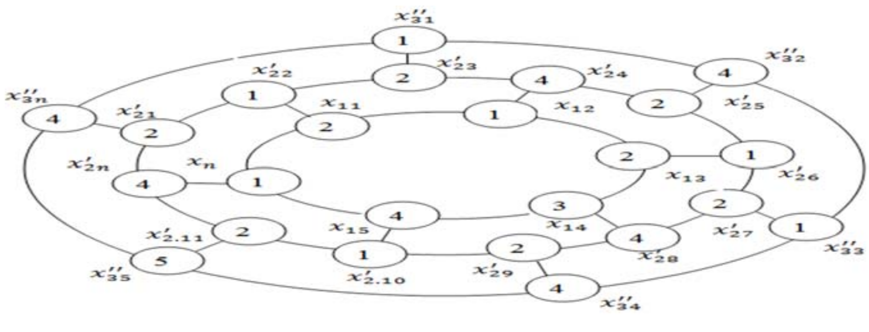

Figure 2.0.2. Coloring with 5 color for $C_{6,12,6}$

The color code for each vertex in $C_{6,12,6}$ with respect to Π are as following:

$$c_\Pi = \big(d(x_n, S_1), d(x_n, S_2), d(x_n, S_3), d(x_n, S_4), d(x_n, S_4)\big) = (0,1,2,1,3)$$
$$c_\Pi = \big(d(x_{12}, S_1), d(x_{12}, S_2), d(x_{12}, S_3), d(x_{12}, S_4), d(x_{12}, S_5)\big) = (0,1,2,1,5),$$

$c_\Pi(x'_{22}) = (0,1,4,2,3)$, $c_\Pi(x'_{26}) = (0,1,2,2,4)$, $c_\Pi(x'_{2.10}) = (0,1,2,1,2)$,
$c_\Pi(x''_{31}) = (0,1,5,1,2)$, $c_\Pi(x''_{33}) = (0,1,3,1,2)$, $c_\Pi(x_{11}) = (1,0,3,2,4)$,
$c_\Pi(x''_{33}) = (0,1,3,1,2)$, $c_\Pi(x_{11}) = (1,0,3,2,4)$, $c_\Pi(x_{13}) = (1,0,1,2,5)$, $c_\Pi(x'_{21}) = (1,0,4,1,2)$, $c_\Pi(x'_{23}) = (1,0,4,1,3)$, $c_\Pi(x'_{25}) = (1,0,3,1,4)$,
$c_\Pi(x'_{27}) = (1,0,2,1,3)$, $c_\Pi(x'_{29}) = (1,0,2,1,2)$, $c_\Pi(x'_{2.11}) = (1,0,3,1,1)$,
$c_\Pi(x_{14}) = (2,1,0,1,4)$, $c_\Pi(x_{15}) = (1,2,1,0,3)$, $c_\Pi(x'_{2n}) = (1,1,3,0,2)$,
$c_\Pi(x'_{24}) = (1,1,3,0,4)$, $c_\Pi(x'_{28}) = (2,1,1,0,4)$, $c_\Pi(x''_{3n}) = (1,1,5,0,1)$,
$c_\Pi(x''_{3n}) = (1,1,5,0,1)$, $c_\Pi(x''_{32}) = (1,1,4,0,3)$, $c_\Pi(x''_{34}) = (1,1,3,0,1)$,
$c_\Pi(x''_{34}) = (1,1,3,0,1)$, $c_\Pi(x''_{35}) = (2,1,4,1,0)$.

The each vertex in $C_{6,12,6}$ is a different color code, then c is the coloring the locating in $C_{6,12,6}$ so we get $\chi_L(C_{6,12,6}) \le 5$. From the proof $\chi_L(C_{6,12,6}) \le 5$ and $\chi_L(C_{6,12,6}) \ge 5$ it can be conclude that $\chi_L(C_{6,12,6}) = 5$.

Case 2. For $n = 7$.

Suppose that the construction of the vertex coloring of a cubic graph $C_{7,14,7}$ is defined by c : $V(C_{7,14,7}) \to \{1,2,3,4,5\}$ such that:

a. $c(x_n) = c(x_{12}) = c(x_{14}) = c(x'_{22}) = c(x'_{24}) = c(x'_{26}) = c(x'_{2.10}) = c(x''_{31}) = c(x''_{33}) = c(x''_{35}) = 1$
b. $c(x_{11}) = c(x_{13}) = c(x_{15}) = c(x'_{21}) = c(x'_{23}) = c(x'_{25}) = c(x'_{27}) = c(x'_{29}) = c(x'_{2.11}) = c(x'_{2.13}) = 2$
c. $c(x_{16}) = 3$
d. $c(x'_{2n}) = c(x'_{24}) = c(x'_{28}) = c(x'_{2.12}) = c(x''_{3n}) = c(x''_{32}) = c(x''_{34}) = 4$
e. $c(x''_{36}) = 5$.

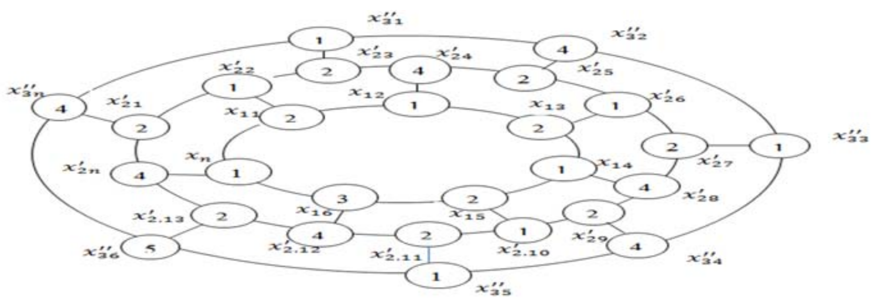

Figure 2.0.3. Coloring with 5 color for $C_{7,14,7}$

Based on this construction, the following color class are obtained:
$S_1 = \{x_n, x_{12}, x'_{22}, x'_{26}, x'_{2.10}, x''_{31}, x'_{33}, x''_{35}\}$,
$S_2 = \{x_{11}, x_{13}, x_{15}, x'_{21}, x'_{23}, x'_{25}, x'_{27}, x'_{29}, x'_{2.11} x'_{2.13}\}$,
$S_3 = \{x_{16}\}$,
$S_4 = \{x'_{2n}, x'_{24}, x'_{28}, x'_{2.12}, x''_{3n}, x''_{32}, x''_{34}\}$,
$S_5 = \{x''_{36}\}$.

From definition of vertex coloring above, it can be seen that each adjacent vertex is color with a different color.

The color code for each vertex in $C_{7,14,7}$ to Π are as following:

$c_\Pi(x_n) = \big(d(x_n, S_1), d(x_n, S_2), d(x_n, S_3), d(x_n, S_4), d(x_n, S_5)\big) = (0,1,1,1,3)$
$c_\Pi(x_{12}) = \big(d(x_{12}, S_1), d(x_{12}, S_2), d(x_{12}, S_3), d(x_{12}, S_4), d(x_{12}, S_5)\big) = (0,1,3,1,5)$,
$c_\Pi(x'_{22}) = (0,1,3,2,3)$ $c_\Pi(x'_{26}) = (1,1,4,0,4)$ $c_\Pi(x'_{2.10}) = (0,1,2,2,3)$,

$c_\Pi(x''_{31}) = (0,1,5,1,2)$ $c_\Pi(x''_{33}) = (0,1,5,1,3)$ $c_\Pi(x_{14}) = (0,1,2,1,5)$,
$c_\Pi(x_{13}) = (1,0,3,2,6)$ $c_\Pi(x'_{21}) = (1,0,3,1,3)$ $c_\Pi(x'_{23}) = (1,0,4,1,3)$,
$c_\Pi(x'_{25}) = (1,0,5,1,4)$ $c_\Pi(x'_{27}) = (1,0,4,1,4)$ $c_\Pi(x'_{29}) = (1,0,3,1,3)$,
$c_\Pi(x'_{2.11}) = (1,0,2,1,2)$ $c_\Pi(x'_{24}) = (1,1,4,0,4)$ $c_\Pi(x_{15}) = (1,0,1,2,4)$,
$c_\Pi(x'_{2n}) = (1,1,2,0,2)$ $c_\Pi(x''_{34}) = (1,1,4,0,2)$ $c_\Pi(x'_{28}) = (2,1,1,0,4)$,
$c_\Pi(x''_{3n}) = (1,1,5,0,1)$ $c_\Pi(x''_{36}) = (1,1,3,0,1)$ $c_\Pi(x_{16}) = (1,1,0,1,3)$,
$c_\Pi(x_{11}) = (1,0,2,2,4)$ $c_\Pi(x'_{2.13}) = (1,0,2,1,1)$ $c_\Pi(x''_{35}) = (0,1,3,1,1)$,

The each vertex in $C_{7,14,7}$ is a different color code, then c is the coloring the locating in $C_{7,14,7}$, so we get $\chi_L(C_{7,14,7}) \leq 5$. From the proof $\chi_L(C_{7,14,7}) \leq 5$ and $\chi_L(C_{7,14,7}) \geq 5$ it can be conclude that $\chi_L(C_{7,14,7}) = 5$.

Case 3. For $n = 8$.

The in Figure 2.0.4. let that the construction of the coloring of the vertex on the cubic graph $C_{8,16,8}$ is defined with $c: V(C_{8,16,8}) \to \{1,2,3,4,5\}$ in such a way:

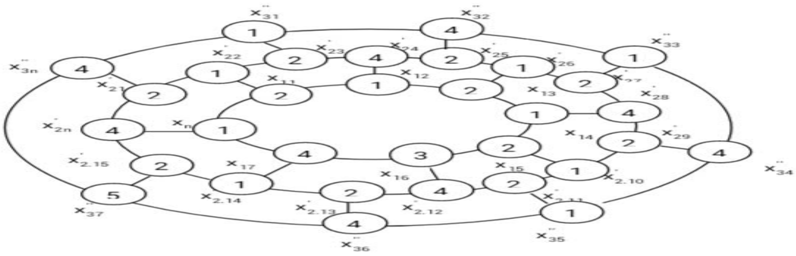

Figure 3.0.4. Coloring with 5 color for C8,16,8.

a. $c(x_n) = c(x_{12}) = c(x_{14}) = c(x'_{22}) = c(x'_{26}) = c(x'_{2.10}) = c(x'_{2.14}) = c(x''_{31}) = c(x''_{33}) = c(x''_{35}) = 1$
b. $c(x_{11}) = c(x_{13}) = c(x_{15}) = c(x'_{23}) = c(x'_{25}) = c(x'_{27}) = c(x'_{29}) = c(x'_{2.11}) = c(x'_{2.13}) = c(x'_{2.15}) = 2$
c. $c(x_{16}) = 3$
d. $c(x_{18}) = c(x'_{2n}) = c(x'_{24}) = c(x'_{28}) = c(x'_{2.12}) = c(x''_{3n}) = c(x''_{32}) = c(x''_{34}) = c(x''_{36}) = 4$
e. $c(x''_{37}) = 5$.

From this construction, the following color class are obtained:

$S_1 = \{x_n, x_{12}, x_{14}, x'_{22}, x'_{26}, x'_{2.10}, x'_{2.14}, x''_{31}, x''_{33}, x''_{35}\}$,
$S_2 = \{x_{11}, x_{13}, x_{15}, x'_{21}, x'_{23}, x'_{25}, x'_{27}, x'_{29}, x'_{2.11}, x'_{2.13}, x'_{2.15}\}$,
$S_3 = \{x_{16}\}$,
$S_4 = \{x_{17}, x'_{2n}, x'_{24}, x'_{28}, x'_{2.12}, x''_{3n}, x''_{32}, x''_{34}, x''_{36}\}$,
$S_5 = \{x''_{37}\}$.

It can be seen in the definition of vertex coloring above, it can be seen that each adjacent vertex is colored with a different color.

There is a color code for each vertex in $C_{8,16,8}$ to Π as following:

$c_\Pi(x_n) = (0,1,2,1,3)$ $c_\Pi(x_{12}) = (0,1,4,1,5)$ $c_\Pi(x_{14}) = (0,1,2,1,6)$,
$c_\Pi(x'_{22}) = (0,1,4,2,3)$ $c_\Pi(x'_{2.10}) = (0,1,2,2,4)$ $c_\Pi(x'_{2.14}) = (0,1,2,1,2)$,
$c_\Pi(x''_{31}) = (0,1,6,1,2)$ $c_\Pi(x''_{33}) = (0,1,5,1,4)$ $c_\Pi(x''_{35}) = (0,1,3,1,2)$,
$c_\Pi(x_{11}) = (1,0,3,2,4)$ $c_\Pi(x_{13}) = (1,0,3,2,6)$ $c_\Pi(x_{15}) = (1,0,1,2,6)$,

$c_\Pi(x'_{21}) = (1,0,4,1,2)$ $c_\Pi(x'_{23}) = (1,0,5,1,3)$ $c_\Pi(x'_{25}) = (1,0,5,1,4)$,
$c_\Pi(x'_{27}) = (1,0,4,1,4)$ $c_\Pi(x'_{29}) = (1,0,3,1,4)$ $c_\Pi(x'_{2.11}) = (1,0,2,1,3)$,
$c_\Pi(x'_{2.13}) = (1,0,2,1,2)$ $c_\Pi(x'_{2.15}) = (1,0,3,1,1)$ $c_\Pi(x_{16}) = (2,1,0,1,4)$,
$c_\Pi(x_{17}) = (1,2,1,0,3)$ $c_\Pi(x'_{2n}) = (1,1,3,0,2)$ $c_\Pi(x'_{24}) = (1,1,5,0,4)$,
$c_\Pi(x'_{28}) = (1,1,3,0,5)$ $c_\Pi(x'_{2.12}) = (2,1,1,0,3)$ $c_\Pi(x''_{3n}) = (1,1,5,0,1)$,
$c_\Pi(x''_{32}) = (1,1,6,0,3)$ $c_\Pi(x''_{34}) = (1,1,4,0,3)$ $c_\Pi(x''_{36}) = (1,1,3,0,1)$,
$c_\Pi(x''_{37}) = (2,1,4,1,0)$.

This that the proof of $\chi_L(C_{8,16,8}) \leq 5$ and $\chi_L(C_{8,16,8}) \geq 5$ can be conclude that $\chi_L(C_{8,16,8}) = 5$.

Case 4. for $n = 9$.

Furthermore, look at Figure 2.0.5 Suppose that the construction of the coloring of the vertex on the cubic graph $C_{9,18,9}$ is determined white $c: V(C_{9,18,9}) \to \{1,2,3,4,5\}$ in such a way:

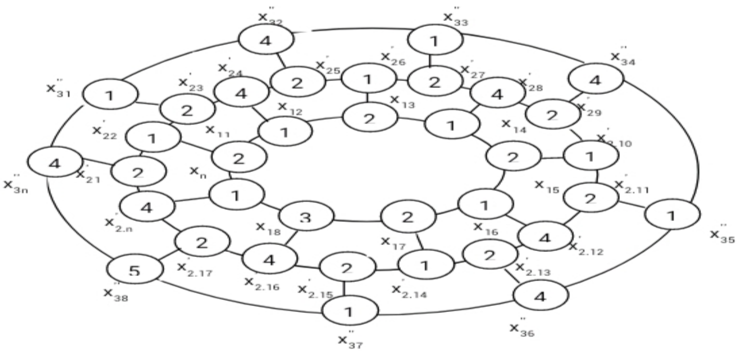

Figure 2.0.5. Coloring with 5 color for $C_{9,18,9}$.

a. $c(x_n) = c(x_{12}) = c(x_{14}) = c(x_{16}) = c(x'_{22}) = c(x'_{26}) = c(x'_{2.10}) = c(x'_{2.14}) = c(x''_{31}) = c(x''_{33}) = c(x''_{35}) = c(x''_{37}) = 1$
b. $c(x_{11}) = c(x_{13}) = c(x_{15}) = c(x'_{21}) = c(x'_{23}) = c(x'_{25}) = c(x'_{27}) = c(x'_{29}) = c(x'_{2.11}) = c(x'_{2.13}) = c(x'_{2.15}) = c(x'_{2.17}) = 2$
c. $c(x_{18}) = 3$
d. $c(x'_{2n}) = c(x'_{24}) = c(x'_{28}) = c(x'_{2.12}) = c(x'_{2.16}) = c(x''_{3n}) = c(x''_{32}) = c(x''_{34}) = c(x''_{36}) = 4$
e. $c(x''_{38}) = 5$.

From this construction, the following color class are obtained:

$S_1 = \{x_n, x_{12}, x_{14}, x_{16}, x'_{22}, x'_{26}, x'_{2.10}, x'_{2.14}, x''_{31}, x''_{33}, x''_{35}, x''_{37}\}$,
$S_2 = \{x_{11}, x_{13}, x_{15}, x_{17}, x'_{21}, x'_{23}, x'_{25}, x'_{27}, x'_{29}, x'_{2.11}, x'_{2.13}, x'_{2.15}, x'_{2.17}\}$,
$S_3 = \{x_{18}\}$,
$S_4 = \{x'_{2n}, x'_{24}, x'_{28}, x'_{2.12}, x'_{2.16}, x''_{3n}, x''_{32}, x''_{34}, x''_{36}\}$,
$S_5 = \{x''_{38}\}$.

this the can be seen of the definition in vertex coloring above, be seen that each adjacent vertex is colored with a different color.

There is a color code for each vertex in $C_{9,18,9}$ to Π as following:

$c_\Pi(x_n) = (0,1,1,1,3)$ $c_\Pi(x_{12}) = (0,1,3,1,5)$ $c_\Pi(x_{14}) = (0,1,4,1,7)$,

$c_\Pi(x_{16}) = (0,1,2,1,5)$ $\quad c_\Pi(x'_{22}) = (0,1,3,2,3)$ $\quad c_\Pi(x'_{26}) = (0,1,5,2,5)$,
$c_\Pi(x'_{2.11}) = (0,1,4,2,5)$ $\quad c_\Pi(x'_{2.14}) = (0,1,2,2,3)$ $\quad c_\Pi(x''_{31}) = (0,1,5,1,2)$,
$c_\Pi(x''_{33}) = (0,1,7,1,4)$ $\quad c_\Pi(x''_{35}) = (0,1,5,1,3)$ $\quad c_\Pi(x''_{37}) = (0,1,3,1,1)$,
$c_\Pi(x_{11}) = (1,0,2,2,4)$ $\quad c_\Pi(x_{13}) = (1,0,4,2,6)$ $\quad c_\Pi(x_{17}) = (1,0,1,2,4)$,
$c_\Pi(x'_{21}) = (1,0,3,1,2)$ $\quad c_\Pi(x'_{23}) = (1,0,4,1,3)$ $\quad c_\Pi(x'_{25}) = (1,0,5,1,4)$,
$c_\Pi(x'_{27}) = (1,0,6,14)$ $\quad c_\Pi(x'_{29}) = (1,0,5,1,5)$ $\quad c_\Pi(x'_{2.11}) = (1,0,4,1,4)$,
$c_\Pi(x'_{2.14}) = (1,0,3,1,3)$ $\quad c_\Pi(x'_{2.15}) = (1,0,2,1,2)$ $\quad c_\Pi(x'_{2.17}) = (1,,1,3,0,2)$,
$c_\Pi(x'_{2n}) = (1,1,2,0,2)$ $\quad c_\Pi(x'_{24}) = (1,1,4,0,4)$ $\quad c_\Pi(x'_{28}) = (1,1,5,0,6)$,
$c_\Pi(x'_{2.13}) = (1,1,3,0,4)$ $\quad c_\Pi(x'_{2.16}) = (2,1,1,0,2)$ $\quad c_\Pi(x''_{3n}) = (1,1,4,0,1)$,
$c_\Pi(x''_{32}) = (1,1,6,0,3)$ $\quad c_\Pi(x''_{34}) = (1,1,6,0,4)$ $\quad c_\Pi(x''_{36}) = (1,1,4,0,2)$,
$c_\Pi(x''_{38}) = (1,1,3,1,0)$.

It can be seen that the proof of $\chi_L(C_{9,18,9}) \leq 5$ and $\chi_L(C_{9,18,9}) \geq 5$ can be conclude that

$\chi_L(C_{9,18,9}) = 5.$

Case 5. For $n = 10$.

And the last note is in Figure 2.0.6. Let that the construction of the coloring of the vertex on the cubic graph $C_{10,20,10}$is determined with $c: V(C_{10,20,10}) \to \{1,2,3,4,5\}$ in such a way:

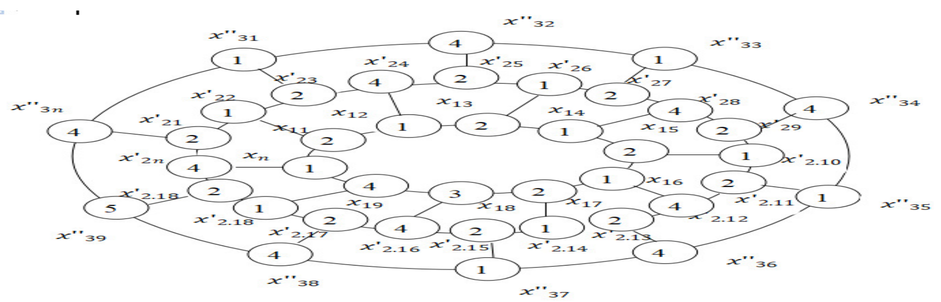

Figure 2.0.6 Coloring with 5 color for $C_{10,20,10}$

a. $c(x_n) = c(x_{12}) = c(x_{14}) = c(x_{16}) = c(x'_{26}) = c(x'_{2.10}) = c(x'_{2.14}) = c(x'_{2.18}) = c(x''_{31}) = c(x''_{33}) = c(x''_{35}) = c(x''_{37}) = 1$
b. $c(x_{11}) = c(x_{13}) = c(x_{15}) = c(x_{17}) = c(x'_{21}) = c(x'_{23}) = c(x'_{25}) = c(x'_{27}) = c(x'_{29}) = c(x'_{2.11}) = c(x'_{2.13}) = c(x'_{2.15}) = c(x'_{2.17}) = c(x'_{2.19}) = 2$
c. $c(x_{18}) = 3$
d. $c(x_{1.10}) = c(x'_{2n}) = c(x'_{24}) = c(x'_{28}) = c(x'_{2.13}) = c(x'_{2.16}) = c(x''_{3n}) = c(x''_{32}) = c(x''_{34}) = c(x''_{36}) = c(x''_{38}) = 4$
e. $c(x''_{3.10}) = 5$.

It can be seen from the construction that the color class are obtained as following

$S_1 = \{x_n, x_{12}, x_{14}, x_{16}, x'_{22}, x'_{26}, x'_{2.10}, x'_{2.14}, x'_{2.18}, x''_{31}, x''_{33}, x''_{35}, x''_{37}\}$,
$S_2 = \{x_{11}, x_{13}, x_{15}, x_{17}, x'_{21}, x'_{23}, x'_{25}, x'_{27}, x'_{29}, x'_{2.11}, x'_{2.13}, x'_{2.15}, x'_{2.17}, x'_{2.19}\}$,
$S_3 = \{x_{18}\}$,
$S_4 = \{x_{1.10}, x'_{2n}, x'_{24}, x'_{28}, x'_{2.12}, x'_{2.16}, x''_{3n}, x''_{32}, x''_{34}, x''_{36}, x''_{38}\}$,
$S_5 = \{x''_{39}\}$.

This it can be seen in the definition of vertex coloring above, it can be seen that each adjacent vertex is colored with a different color. There is a color code for each vertex in $C_{10,20,10}$ to $\prod$ as following:

$c_\Pi(x_n) = (0,1,2,1,3)$ $c_\Pi(x_{12}) = (0,1,4,1,5)$ $c_\Pi(x_{14}) = (0,1,4,1,7)$,
$c_\Pi(x_{16}) = (0,1,2,1,6)$ $c_\Pi(x'_{22}) = (0,1,4,2,3)$ $c_\Pi(x'_{26}) = (0,1,6,2,5)$,
$c_\Pi(x'_{2.10}) = (0,1,4,2,7)$ $c_\Pi(x'_{2.14}) = (0,1,2,2,4)$ $c_\Pi(x''_{37}) = (0,1,3,1,2)$,
$c_\Pi(x_{11}) = (1,0,2,2,4)$ $c_\Pi(x_{13}) = (1,0,4,2,6)$ $c_\Pi(x_{17}) = (1,0,1,2,4)$,
$c_\Pi(x'_{2.17}) = (1,0,1,2,5)$ $c_\Pi(x'_{21}) = (1,0,4,1,2)$ $c_\Pi(x'_{23}) = (1,0,5,1,3)$
$c_\Pi(x'_{25}) = (1,0,6,1,4)$ $c_\Pi(x'_{27}) = (1,0,6,1,5)$ $c_\Pi(x'_{29}) = (1,0,5,1,6)$,
$c_\Pi(x'_{2.11}) = (1,0,4,1,5)$ $c_\Pi(x'_{2.13}) = (1,0,3,1,4)$ $c_\Pi(x'_{2.15}) = (1,0,2,1,3)$
$c_\Pi(x'_{2.19}) = (1,0,3,1,1)$ $c_\Pi(x_{18}) = (2,1,0,1,4)$ $c_\Pi(x_{19}) = (1,2,1,0,3)$,
$c_\Pi(x'_{2n}) = (1,1,3,0,2)$ $c_\Pi(x'_{24}) = (1,1,5,0,4)$ $c_\Pi(x'_{28}) = (1,1,5,0,6)$,
$c_\Pi(x'_{2.12}) = (1,1,3,0,5)$ $c_\Pi(x'_{2.16}) = (2,1,1,0,3)$ $c_\Pi(x''_{3n}) = (1,1,5,0,1)$,
$c_\Pi(x''_{32}) = (1,1,7,0,3)$ $c_\Pi(x''_{34}) = (1,1,4,0,3)$ $c_\Pi(x''_{38}) = (1,1,3,0,1)$,
$c_\Pi(x''_{39}) = (2,1,4,1,0)$.

The can be seen that the proof of $\chi_L(C_{10,20,10}) \le 5$ and $\chi_L(C_{10,20,10}) \ge 5$ can be conclude that $\chi_L(C_{10,20,10}) = 5$.

III. CONCLUSION

In this study, the locating chromatic number of the cubic graph $C_{n,2n,n}$ is the minimum number of color used in the locating coloring of the graph $C_{n,2n,n}$ for $6 \le n \le 10$ obtain the locating chromatic number, namely 5.

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